OPTIMAL DESIGN OF REINFORCED CONCRETE TWO-WAY SLABS

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To

Bā, Mashibā, Kusum and Jayshree

for their patience

CERTIFICATE

I certify that Shri M. C. Thakkar has carried out the work for this thesis, 'Optimal Design of Reinforced Concrete Two-way Slabs', under my supervision and it has not been submitted elsewhere for a degree.

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SYNOPSIS

The optimum design of reinforced concrete slabs with special reference to rectangular two-way slabs supported on walls or stiff beams, is investigated here for various design criteria.

Rectangular two-way slabs on stiff beams or walls designed by conventional methods i.e. 'Elastic analysis - Elastic design', using moment coefficients given in codes of practice, when tested; exhibit excessive safety compared to that for one-way slabs, flat slabs and other flexural members. This excessive safety is due to the evolutionary process by which these coefficients have been developed, and the property of redistribution of forces in the slabs (usually with low percentage of reinforcement). This also influences other basic 'values' in structural design like strength, economy and serviceability. A rational design procedure must take advantage of the redistribution of forces, to aim at desired safety (factor of safety for two-way slabs to kept in par with one-way slabs, flat slabs etc.) and economy; but at the same time satisfying serviceability criteria at working loads.

It is very hard to get an absolute optimal design solution such that all the basic 'values' in structural design like safety, strength, economy and serviceability; are

satisfied simultaneously. Here, in this study optimal solutions are obtained considering each basic 'value' as the governing criterion separately; without using rigorous optimization techniques.

Optimal safety (uniform factor of safety for all flexural members) is attained by the use of Hillerborg's strip method, which is an equilibrium approach in plastic limit analysis; with some modifications for the uniform ereinforcement layouts or with the reinforcement in bands of specified strip widths. Use of yield-line theory is made to analyse the slab to determine the collapse load and the corresponding safety. A separate check for serviceability is made. Optimal strength (uniform strength) design for the given load condition, is obtained by elastic and plastic theories for orthogonal straight but variable reinforcement layout. The behaviour efor serviceability is found to be much improved with these type of layouts, for the uniform thickness of slab. Such layouts (both layouts; by elastic and plastic theories, are different from each other) will show considerable economy for mass production or precast units. Optimal cost design solutions are suggested here by working stress design and ultimate strength design methods, using the moment coefficients specified in codes of practice. A flow chart to get an iterative solution for optimum cost with respect to IS: 456-1964 and prevailing conditions in India; is given.

Hillerborg's approximate theory of elasticity and the concept of 'optimal synthesis are discussed in brief to exhibit their link to the previous chapters. Illustrative design examples are presented here to show the simplicity of the design methods (even for complex slabs). Areas for future research work are also suggested.

CONTENTS

CHAPTER 4: OPTIMAL DESIGN OF TWO-WAY SLABS FOR STRENGTH

CHAPTER 1

INTRODUCTION

1.1 GENERAL

A reinforced concrcte slab forms a very important structural element in the design of roofing, flooring and foundation systems of buildings and bridges. In the case of buildings, the slabs may be subjected to essentially quasi-static loading. Their usual forms are one-way slabs, two-way slabs, continuous slabs on walls or columns with or without beams. They may have complexities like openings, nonuniform thickness and may have shapes such as circular, rectangular, quadrilateral or other geometrical shapes. In the case of bridges they may be beam-floor systems and are subjected to variable loading. In this investigation slabs for buildings under the effects of uniformly distributed loads are of main interest, although the effects of concentrated loads, partition loads and the environment also play an important part in the behaviour of slabs in actual practice. The design of slabs is dependent on their function, size geometry, boundary condition; environmental conditions including loadings, temperature, shrinkage etc; mathematical idealisations of behaviour of slabs for purposes of calculation, and the experience gained and incorporated into the codes of practice. However, as civilisation progresses, there is a need for large covered areas with complex geometry which may involve large spans. Therefore, the need to

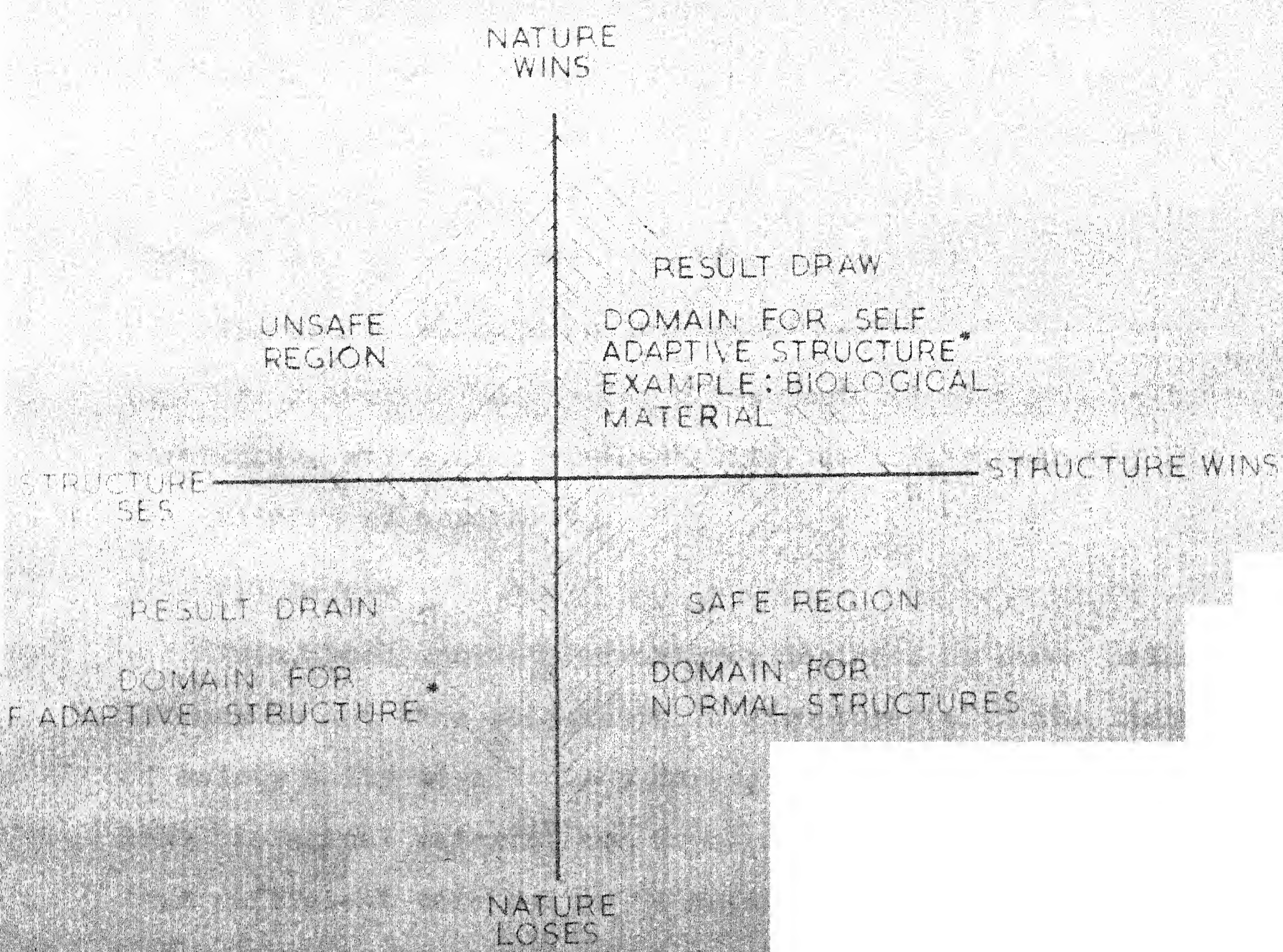
have improvements such as better variety of constituent materials like high strength steel, concrete and increasing use of structural lightweight concrete; is increasingly felt. In the light of the above considerations and a better knowledge of behaviour of reinforced concrete slabs, the limitations of the present codes of practice which tries to link theory and practice and the need for a better understanding of the basic " values", and approaches in structural design is to be studied. This would provide a rationale for introducing changes in codes of practice as rational methods are developed for studying the behaviour of slabs under environmental conditions, dictated by the functional requirements which are divided into design criteria based on basic " values" in structural design.

The role of safety factors through different approaches in structural design and the extent of optimization by the use of codes is significant when optimal design is a specific objective. The concept of " global" optimization as compared to restricted (or local), or optimization with respect to certain optimality criterion on the basis of codes, known herein as " code" optimization is significant, as this gives a feeling of the direction for future research and development. The rationality of different design methods in slabs is important in this respect. Since reinforced concrete slabs are generally ductile (except when there is premature punching shear or bond failure), the use of this behavioural characteristic needs to be increasingly made in the framework of " values" in the

structural design. The remaining sections in this chapter deal with the above aspects in general, with an emphasis on the motivation, need and rationale, for the optimal design of rectangular reinforced concrete two-way slabs.

1.2 BASIC 'VALUES' IN STRUCTURAL DESIGN

Structural design can be defined as the process in which a structure is designed such that it fulfills the needs and can withstand the effect of loads and environment, when a physical situation or a set of situations is given which determine the functional needs, loading patterns and environmental conditions. Structural design may also be defined as a process subject to certain 'problem - solving constraints' so as to design a structure, optimally, subject to certain 'solution constraints'. The problem - solving constraint includes the level of knowledge of the designer, rationality or limitation of codes of practice, time available for design, facilities for computation, experimentation etc. The solution constraint encompass limitations of nature, cost, safety, serviceability, availability of materials, equipment and the nature of construction procedure, supervision and maintenance etc. Structural design can be also defined as a game played by the structure against nature, where the stake is that of failure (functional and/or structural) and their consequence in terms of cost, economy in terms of maintenance over the design life (Refer Fig. 1.1). Both the players (especially the nature) may be



* IF GAME RESULTS IN DRAW, <u>AT ALL TIMES</u>

FIG. III. STRUCTURAL DESIGN AS A GAME BETWEEN NATURE AND STRUCTURE

quite erratic in their behaviour. 'Nature' shows variations in the frequencies and magnitudes of forces like wind load, earthquake forces, environmental effects like live loads, temperature variations, snow and rainfall etc., while the 'structure' shows variations in the frequencies and magnitudes of its material and geometric properties.

The basic 'values' in structural design include safety (uniform factor of safety for all structural elements in the structure), strength, serviceability, ductility, economy and the criteria of optimality.

(i) Safety

Reinforced concrete structures designed by human beings, are not in any case self-adaptive structure (with the factor of safety unity at all times during the life of the structure) like biological material and topological configurations, and thus sufficient care has to be taken to provide reasonable and rational margin of safety, as the problem involves the risk of loss to human life and the effects of failure.[14*]. Moreover the erratic behaviour of the nature compells the design to remain safe even under worst conditions posed by the nature.[14] The use of rational safety factors should be such that a certain structural element should have the same factor of safety as other elements of the same class ~~for~~(e.g. the slabs should have same factor of safety irrespective of the type i.e. one-way, two-way, flat slabs etc.).

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* Indicates the reference number, given in Appendix B.

(ii) Strength

For the worst natural combinations of loads and environmental effects, the structure must have sufficient strength and must give sufficient warning by excessive cracking, deflection etc., before failure. It is also attained due to redistribution of forces in rigidly plastic or elasto-plastic materials. For statically indeterminate structures like reinforced concrete slabs which are ductile, the stress concept underestimates strength considerably for quasi-static loads.

(iii) Economy

Economy and safety go to gether, hand in hand. If structure is more safe, then in general, it can be said that it will be more costly (reverse is not true). But the problem has to be solved with the limited and available funds on the hand~. Thus the optimum values of safety and economy decides the solution of the problem. Economy is very much related to the rationality of codes and the particular optimization criterion considered.

(iv) Serviceability

Structure designed should not show any of the followings at the working loads.

(a) Excessive deflections without the loss of the equilibrium

(b) Premature or excessive cracking

(c) Excessive vibrations, from both a structural and human point of view

(d) Spalling of concrete

(e) Blemishes due to the corrosion of steel or due to the effects of the environments

(f) Excessive deformation in regard to normal use of structure

These abovementioned parameters decide the limit state for which the structure may be designed to assure an acceptable behaviour both at normal (working) loads and at the time of worst combination of loads. The serviceability characteristic consistent with the function of the slab severely restricts the useable strength when high strength steels are used.

(v) Ductility

Ductility in structure, not only provides sufficient warning before failure, but also absorbs considerable energy imparted by earthquake forces, blast forces, impact, shock, moving and other dynamic loads.

Other secondary values are durability, aesthetics, fatigue, fire resistance etc.,Some of the secondary values may become governing design values in certain cases. In the problem of optimal design, the criteria for optimality is important.

(vi) Criteria for Optimality in Structural Design

Optimality criterion may be with respect to uniform factor of safety, strength, economy and serviceability which are the basic design values in the structural design

processes. The most general optimization criterion is that the total cost (initial cost, cost for maintenance, cost of failure multiplied by the probability of failure) should be a minimum.

Here in this study optimality criteria with respect to each of the basic design values (except serviceability) have been considered, separately, for the two-way R.C. slabs.

Optimal safety, as referred hereafter means uniform factor of safety consistent with the other flooring systems like one-way slabs, flat slabs, and other flexural members. Optimal safety satisfies the requirements of the minimum factor of safety (with respect to the collapse load).

While, optimal strength is intended for the concept of uniform strength i.e., the resistance at every point is consistent with the stress field. Thus, optimal strength assures optimal safety, but the reverse is not true.

For economy, the optimality criterion is used in the sense that the initial cost of the slab is minimum.

It will be shown later that it is very difficult to obtain an absolute optimal solution such that all these three criteria are satisfied, yet the separate check for serviceability is to be made.

1.3 APPROACHES TO STRUCTURAL DESIGN

In order to take into account the basic 'values' in structural design and the variations in loads, material properties, the safety factors needs to be incorporated in a rational manner. The main approaches to structural design are:

(1) Deterministic Design

(2) Probabilistic Design

(3) Semi-Probabilistic Design

1.3.1 Deterministic Design

In this, all the design parameters discussed previously are assumed to be constant i.e. without any variations. The highest design loads and the lowest material properties are intended, but because of the cost involved a certain variation is accepted and a factor of safety is assumed to take care of these variations. This approach is followed by most of the codes of practice for the construction material as the reinforced concrete, except 'CEB'(Comite Europcen du Beton) Recommendations for International Code of Practice for Reinforced Concrete[1] and U.S.S.R. Code of Practice[2,3] for Reinforced Concrete.

A simplified structural design process according to the deterministic approach is shown in Fig. 1.2. From Fig. 1.2, decisions are to be taken for various important aspects like loads, analysis, limit states of materials etc. These decisions are taken by an average structural designer with the help of the relevant code of practice for the following.[5]

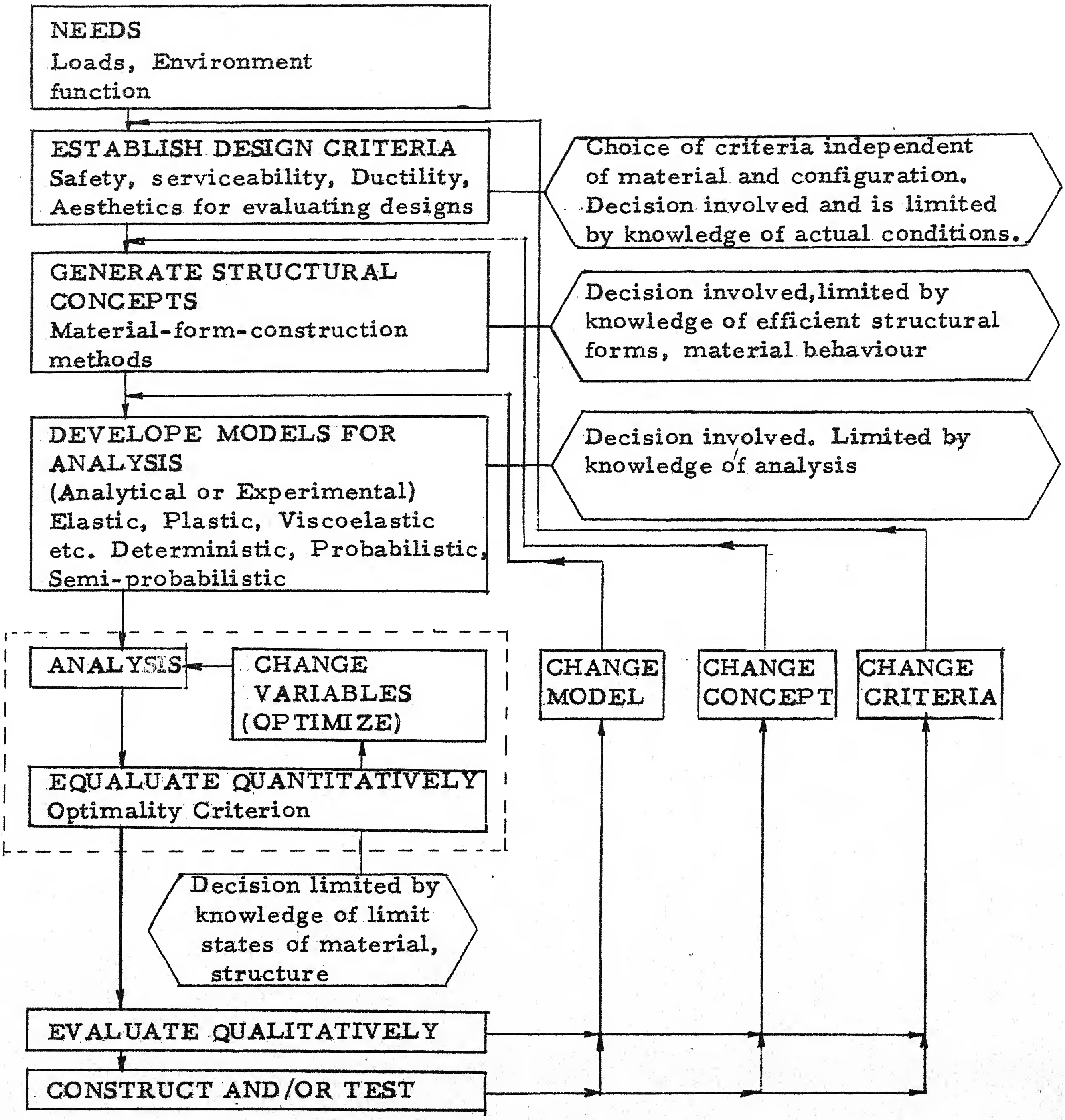
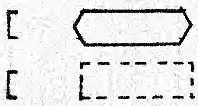


FIG. 1.2: SIMPLIFIED STRUCTURAL DESIGN PROCESS: FLOW CHART (RAO[4])



Decision by Codes of Practice]

Automated Design Using Codes of Practice]

(i) Loads

For the live load as specified in various codes of practice,[6,7,8,9] an analysis by Lind., et al[10], shows considerable variations in the prescribed deterministic values for the design purposes, even for the similar occupancy and function in various countries. Table 1 shows the variations in the live loads specified in the codes of practice, of different countries, for the same function or occupancy. This shows the inherent limitation involved in the value of loads given in the codes of practice.

TABLE 1: VALUES OF LIVE LOADS FOR SAME OCCUPANCY FOR VARIOUS CODES (LIND etal.[10]) (Values in Kg/m^2)

Code	Private Offices	Private Dwelling	School Classroom
U.S.A.	392	196	196
New Zealand	294	147	294
Netherland	250	150	200
Britain	245	147	294
Australia	245	147	294
India	300	200	400
France	200	175	350

(ii) Methods of Analysis

Codes specify static analysis for the dynamic forces like moving loads, earthquake forces, impact and shock, wind etc. by specifying some adhoc value of the 'Impact factor'. Values of this impact factor, are different in various codes.

Moreover, codes specify 'Elastic Analysis' which has been shown by many researchers to be very conservative for slabs regarding safety even with the design carried out in the deterministic format. During the last two decades, considerable progress in the 'Inelastic Analysis' have shown that it is more rational and has taken the place of 'Elastic Analysis' in the codes of practice of countries like U.S.S.R., Sweden[11] etc. However, elastic and visco-elastic analysis are required for serviceability checks.

(iii) Performance Criteria

These may be with respect to deflections, cracking, vibrations, fatigue, fire, environmental effects and these factors are related to the analysis given in the codes for hypothetical loads which relate only qualitatively to the acceptable limit states.

Codes specify working stress design (WSD) and ultimate strength design (USD) methods, with the use of specified loads in codes. WSD has an advantage that it satisfies the serviceability at working loads, but it exhibits in general, excessive safety, and strength at ultimate loads, thus affecting economy. For USD, safety and strength (also economy) are of the values required, but a separate check for the serviceability is necessary. Here use of the plastic behaviour of sections and structures is advantageously made use of, such that slabs are designed for an uniform safety factor as compared with similar flexural members like beams.

The factors of safety, may be either 'global' factors or individual factors to take into account nature of loads, materials and consequences of failure. The latter is more rational since it takes all the factors affecting safety, although, in a qualitative manner. Safety factors as used in codes of practice, limit the optimization with respect to the 'global' optimum.

1.3.2 Probabilistic Design

Deterministic design, which assumes loads, strength of materials, sizes of members etc., to be constant quantities, is not justified because of the varying nature of these parameters. One may find the value of the live load for a residential building from IS:875-1964[12] (or from Table 1), as 200 Kg/m^2, but it can be seen from the occupancy of the floor that the furniture layout, dimensions of rooms, useage etc., are not fixed and this leads to very less magnitude of live load (equivalently uniformly distributed) for most of the time, during the entire life of the structure. Also, it is a function of socio-economic factors like the location of the place in a region. Codes do not take advantage for safety by specifying rational loads with a provision that the floor may be overloaded (though safe) some times in the entire life of the structure. Thus, codes require higher factor of safety (to be more correct, the 'factor of ignorance') by making provisions for far increased loads and decreased strength of materials.

It is necessary for providing rationally safe design, to study the probability of having overloading or reduced strength, and therefore probabilistic design, where probability of failure (probability of having loads more than resistance) or reliability (probability of having resistance more than loads) are considered; is purely a statistical approach. More rigorous treatment on probabilistic design can be found elsewhere.[13] Using the probability of failure. Sawyer[88] related the factor of safety (the inverse of the ratio, load/ general collapse load); the cost including loss upon the failure and (Fig. 1.3) probability of failure. It is seen that as the probability of failure is decreased, the loss as compared with the cost of building increases. This also requires distribution of variations for the various design parameters. Data for the variations in loadings is at present not available or very scanty. Also because of control in design and execution all parameters are not necessarily random. Considering this limitation of the probabilistic design, a compromising approach using advantages of both deterministic and probabilistic design approaches, is made by Russian code and CEB code. In absence of any specific name given to this approach, it is called hereafter as 'Semi-probabilistic' design approach. It is also called as the 'limit state design' approach.

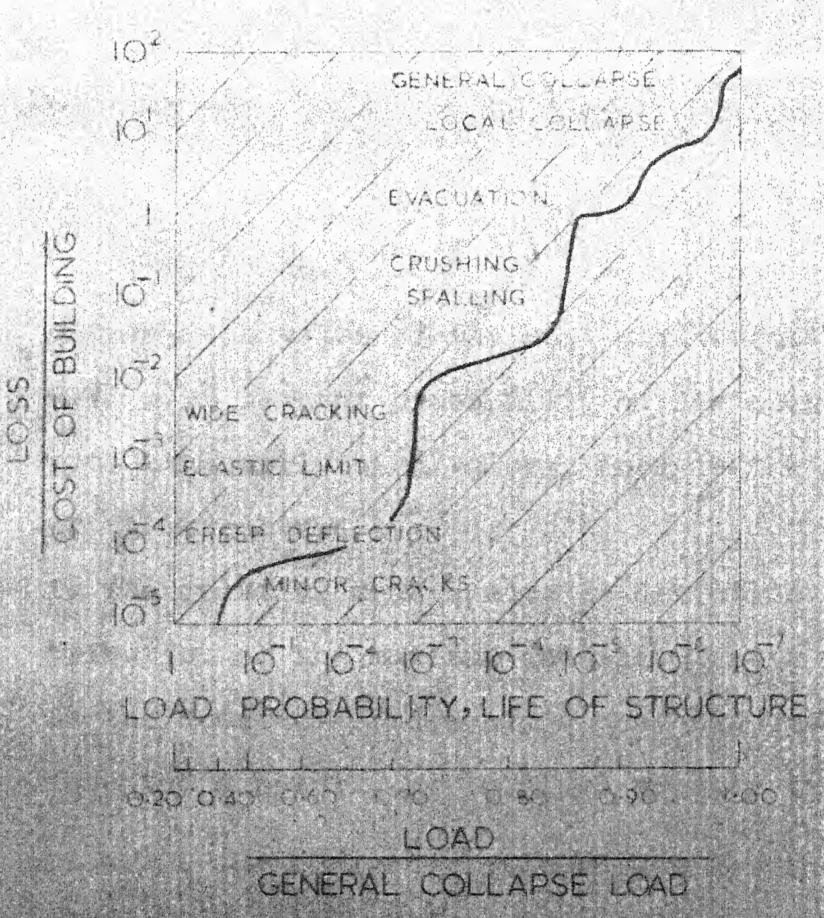


FIG.1.3 PROBABILITY OF FAILURE VS. LOAD VS. COST RELATION FOR A R.C.C. STRUCTURE (SAWYER)(88)

1.3.3 Semi-probabilistic Design

Statistical variations in the loads and material strength are considered, here by the concept of 'characteristic' values which define in a probabilistic format, the maximum loads and the strength of the constituents in a rational manner. In this individual safety factors are attached to get design loads and strengths.

Characteristic value of the load (Q_k)* can be defined as:[1]

$$Q_k = Q_m (1 + \overline{K} \delta) \qquad \ldots \quad (1.1)$$

where Q_m denotes the value of the most unfavourable loading, with a 50 percent probability of its being exceeded, upto abnormally high values, once in the expected life of the structure.

δ is the relative mean quadratic deviation of the distribution of the maximum loading.

$\overline{K}$ is the coefficient depending upon the probability, accepted a priori, of loadings greater than Q_k.

Similarly, the characteristic values for the strength of materials (σ_k) can be defined as:

$$\sigma_k = \sigma_m (1 - \overline{K} \delta) \qquad \ldots \quad (1.2)$$

Where σ_m is the arithmetic mean of the various experimental results.

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* Appendix A lists the notations followed.

$\bar{K}$ is the coefficient depending upon the probability accepted a priori, of the test results being below the value σ_k and also on the number of tests definin σ_m.

In Equations (1.1) and (1.2) concept of probabilistic design is made use of. Now, the design values for the load and strength of materials can had by increasing or decreasing with the help of some factor γ, as shown below.

<u>Design load and strength</u>:

$$Q^* = \gamma_s \; Q_k \qquad \ldots \quad (1.3)$$

and

$$\sigma^* = \sigma_k / \gamma_m \qquad \ldots \quad (1.4)$$

where Q^* and σ^* are design load and design strength respectively,

γ_s and γ_m are the factors are chosen considering the following parameters.

Parameters affecting γ_s (Safety factor with regard to strength):

(i) The probability of a limit state

(ii) The possible increases in the permanent or superimposed loads

(iii) Accuracy of the assumptions accepted for the design and accuracy of design calculations.

(iv) Possible errors in construction

(v) Probability of worst combination of several loadings

(vi) Possibility of redistribution of stresses

(vii) Importance of the risk of human life.

Different factors for γ_s are given for dead load and superimposed constant live loads, varying live loads and dynamic loads.

Parameters affecting γ_m (Safety factor with regard to material strength):

(i) Quality control at site (i.e. manufacturing conditions)

(ii) Tolerances in dimensions of members

(iii) Effects of prolonged or long-term loading i.e., rate of loading, size and shape effects

(iv) Warming before failure

The value of γ_s is different for different limit state i.e., failure, deflection, cracking and spalling at working loads.

The product of γ_m and γ_s is an index of the total safety and is dependent upon the probability of failure.

Having known the design loads and strength of materials from Equations (1.3) and (1.4), for a given probability of failure, one can very easily thereafter design the structure using the deterministic approach, which is quite palatable to the average structural designer. Thus, semi-probabilistic limit state design approach is an ideal combination, where advantages of both, deterministic and probabilistic approaches are taken.

1.4 CODES AND THEIR EFFECT ON SAFETY AND ECONOMY

For the code of practice, the theoretical factor of safety depends upon the hypothetical loads and corresponding performance criteria. But the actual factor of safety is dependent on true (and yet may be unknown) behaviour and exact methods of analysis. Thus excessive safety shown for the design according to the code is due to hypothetical loads and approximate methods of analysis, which do not take into account the reserve strength due to redistribution of the forces, for the elasto-plastic material like reinforced concrete with low percentage of reinforcement, and the presence of arching and other membrane action.

Code provisions with the basis discussed above, are, in general, very conservative with respect to economy also. Lind, etal,[10] commented that in Canada alone due to the conservative provisions of the National Building Code, estimated losses to the society are to the tune of 10^5 to 10^7 dollars a year. This stresses the far-reaching effects of codes for safety and economy.

These anamolies in the codes are due to the fact that the design according to a code is an optimization only to the extent that the design of the code in the first place was an optimization based on what was known at that time.' Also, it can be seen from Fig. 1.2 that the design according to the code can lead to 'restricted' (or constrained) optimum rather

than 'global' (or unconstrained or free) optimum which can be obtained by a thorough search and research in the structural design process (Fig. 1.2) for the hidden element of rational safety i.e., true safety factor and a quantitative assessment of the factor of safety required in a particular design.

Conservatism in the codes can be removed to certain extent by specifying rational provisions at least taking full advantage of the redistribution of forces and waiving the prevailing uncertainties regarding loads and performance criteria. The basis in the code for last two parameters is the factual knowledge gained from the inconclusive scientific data on loads and behaviour of actual full size structures for the entire life span of the structure, and the observation for the performance criteria of the structure designed on the basis of data of full-size tests. Thus, code provisions are specified using the evolutionary process on the previous provisions rather than performance oriented specifications with respect to basic 'values' in structural design. This would help in the full utilization of knowledge of behaviour of structures like slabs in design practice and help the trend towards improvement in materials science and technology.

1.5 CONSERVATISM IN THE CODES FOR THE DESIGN OF THE TWO-WAY R.C. SLABS.

Short-term loading tests carried out by various research workers on the two-way R.C. slabs designed by the conventional

moment coefficients prescribed by the codes, have indicated excessive safety with respect to the ultimate load carrying capacity of the two-way slabs as compared to one-way slabs and flat slabs. This excessive safety, observed in the tests is due to various reasons like presence of membrane action, strain-hardening of the steel and redistribution of forces. Present provisions of the codes neglect these beneficial effects. It is really very difficult to take into account the membrane action (especially tensile membrane action) and the strain-hardening effects in the design procedures due to complexities involved in incorporating them, however redistribution of forces (a quality due to elasto-plastic behaviour of the R.C. slab with low percentage of reinforcement) can be taken into account for the design of the two-way slabs. Moment coefficients prescribed in the various codes for the design of the two-way R.C. slabs have been evolved using the mathematical theories of elastic plates. Thus, the conservatism in the design of the two-way R.C. slabs, is a function of the evolutionary process for design procedures specified in the codes and the developments in plastic analysis of structures and the knowledge of actual behaviour of slabs.

The evolution of design procedures in the codes for various flooring systems has an influencing role for the resulting effects on the basic values discussed previously. Compared to the two-way R.C. slab, another flooring system, known as flat slab (with or without drop panels) is quite popular in

many countries. Its popularity is due to a major factor other than elegence, aesthetic excellence, reduction in effective storey height, ease in the construction etc. This major factor is the rational safety and economy in the design of flat slabs as compared with two-way slabs. It is quite evident from the preceeding discussions that the safety influences other basic values also. Searching the reason for the rational safety in the case of flat slab, one finds this is due to the fact that the construction of flat slabs preceeded the knowledge of its analysis. Safe and successful completion of flat slabs (where full redistribution of forces is exploited) showed acceptable behaviour both at the working and ultimate loads. As compared to the flat slabs, the two-way slabs were developed by the mathematical theory of elasticity and it was designed by the conventional straight line theory of working stress design which neglects tension in the concrete. However, for usual working loads, for R.C. members with low percentage of steel, the working load is in the uncracked region. Later on, with the developments in the plastic limit analysis of slabs it is experimentally verified that their factors of safety are much higher as compared to beams one-way slabs and flat slabs. None of the elastic theories for the two-way slab do take full advantage of redistribution and aim at uniform safety for all flooring systems. Sozen and Siess,[15] have pointed out that the safety in the two-way slabs is more by 67 % than the flat slabs, by making comparative study of the total design

moments for the typical interior panel. The American Concrete Institute code (ACI) for its 1970, version is rectifying the above anamolies.

The prevailing conservatism in the design of the two-way slabs, has given an impetus to the search for the rational design theory, such that the factor of safety (with respect to ultimate load) for the two-way slabs is kept in par with that for one-way slabs, flat slabs and other flexural members. And many theories which were developed in last two or three decades (as can be seen from Fig. 1.4) (detailed discussion, for the advantages and limitations of each theory is discussed in the next chapter), have failed to satisfy this basic requirement of the structural design process i.e. uniform safety factor.

Thus, the requirements of the rational design theory are:

(i) to provide safety consistent with that of one-way slabs, flat slabs etc.

(ii) to take full advantage of the reserve strength by considering full redistribution of forces i.e. by using appropriate plastic methods of analysis and design.

(iii) to assure the satisfactory performance at the working loads.

(iv) to provide a very simple format, such that an average designer can use it without any trouble of finding out worst combination or pattern loading

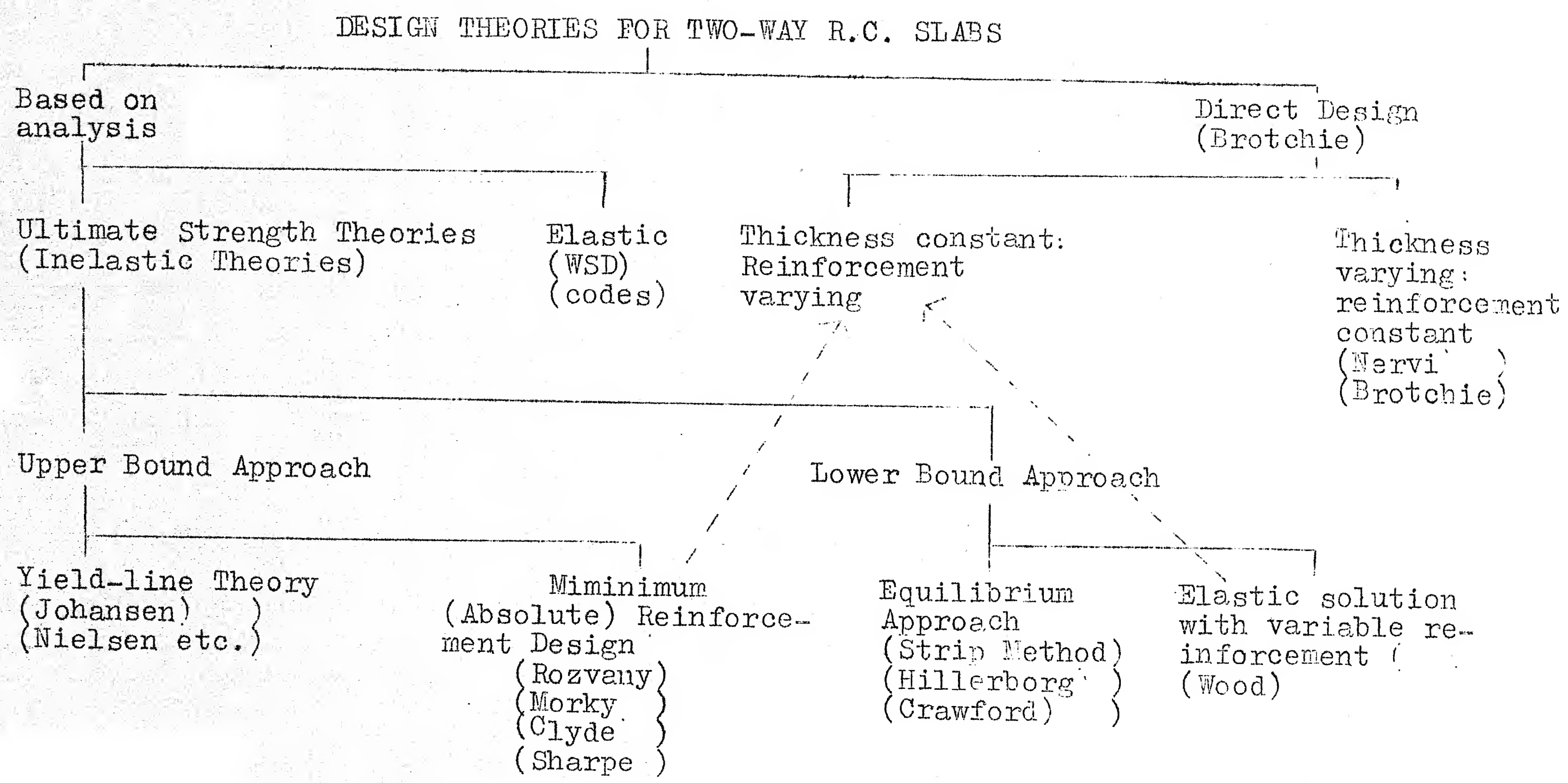


FIG. 1.4: VARIOUS DESIGN THEORIES FOR TWO-WAY R.C. SLABS

(which occur in practice rarely) for the computation of maximum moments at critical sections, and

(v) to allow the use of various concrete mixes (including structural light-weight concrete) and various types of steel (including high strength steels, deformed bars etc.).

1.6 OBJECT AND SCOPE

The main object of the study is to obtain optimal design of reinforced concrete rectangular two-way slabs for different criteria and to explore the possibility of application of Hillerborg's strip method for the optimal design of such slabs and other complex slabs. The use of redistribution of moments for taking advantage of the plastic behaviour by means of the Equilibrium method such that it is useful for getting design coefficients is the main aim, although for completeness; studies for uniform strength by assumption of variable reinforcement, studies for economy of design of such slabs are secondary objectives.

The search for an optimal solution for the design of the two-way R.C.rectangular slabs, for each of the basic values mainly safety, strength and economy, separately carrying out, (leading to 'restricted' optimum rather than 'global' minimum) is the objective of the study, since an optimal synthesis of all these factors is complex. Considering the conservatism

prevailing in the design of the two-way R.C. slabs, as discussed in the previous section, simplified design procedure (though rational from the safety viewpoint). With the help of lower bound approach (using full redistribution of forces) satisfying all requirements discussed in the previous section, is to be explored and suggested, for the improvement in the codes, such that the safety in the two-way slabs is consistent and kept par with one-way slabs, flat slabs and other flexural members in R.C. construction.

Two-way slabs for precasting work or for repetitive type of work e.g., mass housing projects, require careful study for the placing reinforcement most effectively and more economically. This leads to the slab of uniform strength, where the thickness of the slab is kept constant (by some criteria like stiffness, economy etc.) and the reinforcement layout is kept varying from point to point in the slab by either elastic theory or optimum plastic design. Such a reinforcement layout is known to exhibit acceptable behaviour both at a working loads.

For the known field of moments, various design solutions (by either elastic theory i.e. working stress design or ultimate strength theory) using different thicknesses and amount of the reinforcement are available for the use. It is intended in this investigation to find out in most general form, the economical combination of the overall thickness of the two-way slab and the amount of the reinforcement.

With the help of the approximate theory of elasticity (after Hillerborg[16]), a simplified design procedure is introduced to an average structural designer, for the complex slabs i.e., the two-way slabs supported by an intermediate column or columns, or wall or walls, or re-entrant corners.

In chapter 2, a critical review of the research work done in the area of optimal design of two-way slabs is made.

A rational design procedure for an optimal (optimal in the sense that the safety is in par with the one-way slabs, flat slabs etc.) safety for various moment distribution across the panel for various codes, is suggested in chapter 3, by using the lower bound approach of Hillerborg's strip method[17], after some modifications to the strip method. Conservatism in the codes, regarding safety is also focussed in this chapter. Finally, the moment coefficients for the working load, based on working stress design, are suggested, which satisfy the serviceability requirements by stiffness requirements etc.

Minimum reinforcement layouts for the panels with different boundary conditions by the elastic theory and optimal plastic theory are studied in chapter 4. Here, the comparisons are made to the values of amount of the reinforcement required by the other theories, codes etc. to find out the amount of the conservatism involved for the two-way slab as compared to the two-way slab with uniform strength (or an optimal strength). A direct design approach (after Brotchie[18]), without assuming

the proportions of the slab, is also introduced, in this chapter.

Optimum (minimum) cost solutions for the elastic theory using the coefficients specified in the codes or those obtained in chapter 3 by the rational design procedure, and ultimate strength theory, are searched for, in the chapter 5, by an exhaustive iterative search and the classical theory of differential calculus, respectively. Some solutions for minimum cost, for conditions in India, are also presented.

Chapter 6 introduces Hillerborg's approximate theory of elasticity[16] for the simplified direct design of the complex slabs. Hillerborg's rules[16] for the termination of the reinforcement are also introduced for the quick reference, in a simple form.

Illustrative design examples showing the advantages of the rational design theory over the codes are included in Chapter 7. This chapter also shows simplified design procedures for complex slabs.

'Restricted optimum' is searched in chapter 3 to 5, by considering each corresponding basic design 'value' separately. But the 'Global optimum' can be had by considering all the basic 'values' in the design process to be effective. Such a process, branded as an 'optimal synthesis' is presented in chapter 8, with due regards to the difficulties encountered in the search for the 'global optimum' solution.

In chapter 9, critical remarks on the study, conclusions from the study and focussing the limitations of the approaches considered in chapter 3 to 5 are made alongwith the recommendations for the areas of future research, where the state of knowledge is absolutely poor or inconclusive or unknown.

CHAPTER 2

LITERATURE SURVEY

2.1 'THEORY OF ELASTICITY' AND 'TWO-WAY SLABS'

As stated in the previous chapter, the prevailing conservatism in the present design procedures is due to the mathematical development of the design procedure for the two-way R.C. slabs. Construction of flat slabs is carried out to suit the load tests and experimental evidence, while the two-way slabs were constructed to suit the then design methods (obviously based on the elastic analysis neglecting the tension in concrete).

Lagrange in 1811, presented the equilibrium equation of the plate (in terms of deflection). Navier in 1850 was first to solve the equilibrium equation presented by Lagrange, for the very special case of an uniformly distributed loads over the simply supported rectangular plates. Levy's and Rayleigh-Ritzs' solutions were later available for the plate with other type of boundary conditions. Coefficients suggested by Rankine-Grashof (based on usually known as cross-stick method) were for the plate allowing corners to be lifted up. Here, they considered the compatibility of the deflection at the centre of the slab, to derive the moment coefficients. Later on Marcus corrected the Rankine-Grashof's coefficients for the torsion, when the corners are not lifted up.

Similar to the coefficients given by Rankine-Grashof (after the correction for the torsion by Marcus), design moment coefficients are given by others like Westerguard[19], Pigaud, Rogers[20], Czerny[21], etc.

These all coefficients using the theory of elasticity considering the worst combination of loadings or pattern loadings, have been adopted in the codes of practices of various countries except USSR.[2,3] IS:456-1964[7] Method 2 (more rational than Method 3) has been based more or less on Czerny's[21] coefficients. DIN:1045[9] also recommend the use of Czerny's[21] coefficients. ACI:318-63[6] Method 3 (most rational than Method 1 and 2) is based upon the coefficients suggested by Rogers.[20] Moment coefficients specified by the CP:114-1957[8], are more or less the same as proposed by Rankine-Grashof with Marcus's correction.

All these moment coefficients have shown an excessive safety with respect to the ultimate load. This inconsistency and the inelastic behaviour of the two-way slabs with low percentage of reinforcement led to the development of the 'inelastic design theories', using the upper and lower bound theorems of the limit analysis, as 'yield-line theory' and 'Hillerborg's strip method', respectively.

2.2 YIELD-LINE THEORY

In 1920, Ingerslav[22], while testing the two-way slabs to the failure, observed that slabs at failure appeared to be

divided into segments by well defined crack patterns which could reasonably well be idealized as straight lines (rupture-lines or fracture lines).

Johansen[23] in 1930 developed Ingerslav's concept with few modifications to account the effects of shear and twisting moments along the yield-line. Present form of yield-line theory is due to Johansen.[23]

Assumptions of the yield-line theory are enumerated as below:

(1) The reinforcing steel is fully yielded in the yield lines at failure.

(2) The slab is divided into linear (neglecting elastic deformation) segments at the failure.

(3) Johansen's[23] stepped yield - criteria is satisfied (refer section 3.4.5).

(4) Moments are uniformly distributed along the yield-line

Yield-line theory has certain limitations as described below:

(1) It is very good and simple method, but it can be used as a design method for the case of isotropic reinforcement only. For the slabs with orthotropic reinforcement, this method can be used as a method of analysis only.

(2) Many alternative yield-line patterns have to be considered to arrive at the correct value of the collapse load. Thus, even for the analysis this method becomes very complicated and cumbersome to an average structural designer.

(3) Unless and otherwise the correct yield-line pattern is found, it will give unsafe value of the collapse load.

(4) As a method of analysis, it is unsuitable for analysing accurately, the slabs with non-uniform reinforcement or curtailed reinforcement.

(5) The design by yield-line theory many times may result in an uneconomical solution.

With all the above limitations and assumptions, yield-line theory provides very powerful tool for analysing the slabs. This can be seen from its popularity in European countries. However, as a design theory, being an upper bound approach, it will give the unsafe value of the collapse load. Such tests are available (incorporating membrane action and strain-hardening effect) to indicate that the yield-line theory gives the conservative value of the collapse load.

Nylander[24], Zaslavsky,[25.26] Nielsen[27] etc. have critically studied and used the yield-line theory in their research work. 'European Comite du Beton'(CEB), have published bulletins from time to time advocating and illustrating the use of the yield-line theory.[28,29,30] Jain and Jain[31], studied the deflection

considerations in the slabs with one edge free, designed by the yield-line theory. They conclude from the experimental and theoretical results, that the yield-line theory or any other inelastic theory should not be used when one or more supporting edges are free. Parkhill[32] showed from the lower bound solution for simply supported square slab under uniformly distributed load based on the assumption of the elasto-plastic behaviour of the slab material, that the elastic moments within segments of chosen yield-line pattern are statically admissible everywhere. Taylor[33], Jacobson,[34] Morley[35] etc., have extended the conventional yield-line theory by considering both compressive and tensile membrane action.

Recent advances in yield-line theory i.e., nodal-force theory, incorporating membrane action, fan pattern etc. have been reported elsewhere.[36] Massonet[37] developed criterion for kinematic admissibility of pattern of yield-lines and for static admissibility of field of moments, distribution of moments and support reactions without using nodal force theory. Save[38] connected various approaches to nodal-force theory. Janas[39] has discussed kinematic compatibility requirements for the slab with isotropic reinforcements with and without membrane action. Jones and Wood[40] have recently collected all the information on the yield-line theory in their book. Petcu[41] has studied the influence of exactness of parameters defining yield-line pattern, on the plastic moment values or the collapse load.

During last two decades, as can be seen from the interest of research workers in the yield-line theory, in making the theory very sophisticated by considering membrane action, nodal-forces and their effects, fan patterns etc. Thus, the original simple form of the yield-line theory (essentially a method of analysis) is lost due to considerable improvements to make it more rational. An average structural designer will not be in a position to apply the yield-line theory of today due to its complexities and cumbersomeness.

2.3 HILLERBORG'S STRIP METHOD

To ensure safety, Hillerborg[16] in 1960 proposed a strip method which is based on the equilibrium approach (lower bound approach). Hillerborg's strip method assures the conservative or at least required value of the collapse load. Detailed discussion on this method can be found in chapter 3. For the two-way slabs supported by an intermediate support of column or wall or re-entrant corner, Hillerborg[17] proposed the approximate theory of elasticity, details of which can be found in chapter 6. Crawford[42,43] was the first to critically evaluate Hillerborg's strip method and approximate theory of elasticity. Crawford[42] observed for the case of simply supported square slabs, that Hillerborg's strip method offers a very poor lower bound on the collapse load. Wood and Armer[44] have examined Hillerborg's strip method in the light of the strict limit analysis regarding the discontinuity and the orientations of

zero shear lines. In spite of all the criticisms Wood and Armer have made on this method, they are taking out the book[45], which uses this method for designing slabs. Armer[46] has reviewed the work of Hillerborg', very recently.

Thakkar and Rao[47], using the strip method with some valid modifications, have recommended moment coefficients for the design of the slabs according to other provisions of IS:456-1964[7](similar to method 2), assuring the optimal and rational safety (details can be found in chapter 3) for the uniform moment distribution without the use of torsion reinforcement. Kemp[48] provided a correct lower-bound solution for the simply supported square slab such that the yield criteria for the R.C. slabs will be satisfied.

Kemp[49] in some other paper, has proposed the rational (though not fully correct, because of the presence of the membrane action and strain-hardening of reinforcement) yield criteria such that the normal and tangential moments along the yield-line should be both less than corresponding values of the stress-field. Massonet[50] recently introduced somewhat more rational yield criteria, incorporating strain-hardening. Johansen's[23] stepped yield criteria and Nielsen's[27] linearised criteria are essentially same. If Mise's, Tresca's or Square yield criteria is used for the R.C. slab, it will provide very conservative value of the collapse load comparated to that observed during tests.

2.4 OPTIMAL STRENGTH DESIGN

Like the beam of uniform strength, the slab with uniform strength provides an optimal strength design as proposed by Brotchie[18] in more general form. Moment in the R.C. slab can be assumed as the function of the overall thickness of the slab and the area of reinforcement in the direction of the moment. If the reinforcement is kept uniform all over the plate, then for fully stressed (either by ultimate strength theory or working stress design) condition, one can use theory of elasticity to derive the thickness of the slab varying from point to point by using 'Direct Design' concept of Brotchie[18]. But when the thickness of slab is kept uniform (found from the stiffness or economic considerations), then the reinforcement layout is varying from point to point and this provides minimum reinforcement volume for the slab.

2.4.1 Minimum Reinforcement Design by Elastic and Plastic Theories

Using the concept of Hillerborg[51] for the economical layout of reinforcement, Wood[52,53] established the orthogonal straight reinforcement layout for the slab with uniform thickness, satisfying the elastic stress field everywhere in the slab, is 'near' to the absolute optimal solution. Rozvany[54,55,56] and Morley[57] waiving the requirement of the codes for the minimum reinforcements at the point of zero moment and using the plastic theory derived an absolute optimal solution for the simply supported square slabs for both constrained and unconstrained solutions. Rozvany uses the virtual displacement field which satisfies

Heyman's[58] theorem and the kinematic requirements as specified by Prager and Shield[59].

Clyde and Sharpe[60] optimized the two-way slabs for various boundary conditions except all sides clamped, using Hillerborg's strip method. Brotchie [61] uses his 'Direct-Design' approach to suggest rational design theory with reinforcement layout varying from point to point.

Rozvany[62] shows nearly 70 percent economy in the reinforcement for the circular footing by using orthogonal curvilinear reinforcement layout. Kalizsky[63,64] and Charret[65] using the quadratic variation for moment, take into account the requirement of minimum reinforcement for environmental effects.

Recently, Brotchie[66] used maximum potential energy criterion to drive minimum reinforcement layout which satisfies other criteria implicitly. like maximum stiffness, maximum or minimum strain energy, minimum weight, uniform strength etc.

2.5 OPTIMAL COST DESIGN

For the optimal cost design, Thakkar and Rao[67] suggested an iterative procedure using the moment coefficients specified in the codes for the design by working stress method. This iterative procedure (details can be found in chapter 5) also satisfies the constraints of the codes, such as the minimum thickness of the two-way slab, the minimum percentage of reinforcement, the minimum cover and the maximum spacing between two bars.

Traum[68] uses the classical method differential calculus to get optimal cost parameters, the reinforcement percentage and then the effective depth of the slab. Traum's[68] approach, of course, yields good results for the simply supported rectangular slabs, but fails miserably for the continuous slabs. Norman[69] practically uses the concept of Traum[68], except that the considerations for unit costs of the ingradients of the concrete are made. Norman concludes that the least cost concrete is the one with the leanest (so obvious) concrete (in India M.150). Traum's[68] and Norman's[69] approaches are for the ultimate strength theory.

Thakkar and Rao[70] suggested the moment coefficients using Hillerborg's coefficients, which can be used for the design by working stress method for the optimal solution. These are presented in chapter 4.

Kalizsky[64] derived, using the principles of minimum strain energy, that the least cost slab is the one in which reinforcement layout is in accordance to the elastic stress field.

2.6 EXPERIMENTAL AND COMPARATIVE STUDIES

Experimental investigations to find out the exact behaviour of the two-way slabs had been carried out, by many research workers. Dresden (1930) and Stuttgaret (1925) tests are reported in reference (28). University of Illinois team of Siess, Sozen etc.[71,72] have tested many full size two-way slabs in nine Sq. panels, supported by stiff beams to verify the effects of

pattern loadings on cracking, deflection etc. and to explore the conservatism of the design according to the relevant code. Casillas and Siess[73] have made comparative studies of all design methods of the ACI code at that time and one suggested by Rogers[20]. Ahuja[74] reported the results of full scale tests on two, one-way and one, two-way slabs, designed according to IS:456-1964.

Armer[75] tested some two-way slabs designed by Hillerborg's strip method and approximate theory of elasticity. Taylor et al[76], studied the effect of the arrangement of reinforcement on the behaviour of two-way R.C. slabs.

Rozvany[56] studied the behaviour of the simply supported square slab with absolute minimum reinforcement layout confirming requirements of optimal plastic design and other with uniform isotropic reinforcement designed by the yield-line theory. Rozvany shows that the behaviour of optimized slab is better than the one with an uniform isotropic reinforcement.

2.7 WORK IN THE RELATED AREAS

Work in the related areas and optimization of structures (particularly plate structures) have been reported in references given in Appendix C as Bibliography (over and above those given in Appendix B).

CHAPTER 3

OPTIMAL DESIGN OF TWO-WAY SLABS FOR SAFETY

3.1 INTRODUCTION

In this chapter, the optimal design of two-way slabs for safety is considered in the sense that the factor of safety of two-way slabs is considered in par with that for beams, one-way slabs, flat slabs by the use of plastic limit analysis. The lower bound theorem of limit analysis i.e. Equilibrium method as modified by Hillerborg is used so that it is useful for getting design moment coefficients. However, 'global' optimization is possible when rational safety factors, as discussed in chapter 1, are used.

Reinforced concrete two-way rectangular slabs, being highly redundant structural systems, when analysed by the theory of elasticity and designed by working stress design method (WSD) for uniformly spaced reinforcement without curtailment of bars; have been known to give higher factor of safety when tested to failure. This becomes more so for the two-way slabs as compared to the one-way slabs. A case in point, is the testing of two, one-way slabs and one, two-way slab designed by IS:456-1964, by the National Building Organisation[74], yielding factors of safety of 3.7, 3.25 and 7.47 respectively. The anamolies, in the design of multiple panel floor slabs and flat slabs according to the codes of practice in various

countries, based on test results to failure, show that the factor of safety of the two-way slabs is higher than the flat-slabs. This is because the design methods, as discussed earlier, for the two-way slabs were developed by the mathematical analysis of elastic plate equations and the use of working stress design neglecting tension in concrete (which is considerable for members with low percentage of reinforcement); while the flat slabs were built using load tests even before the methods of analysis were developed. Sozen and Siess,[15] showed that the factor of safety for the two-way slabs is about 50 percent higher than that for the flat slabs, by making comparative study of total design moments per panel with different boundary conditions and span ratios.

It may be interesting to note that ACI-ASCE Committee 421 have recommended changes in the design of two-way slabs so that the factor of safety is in parity with that of the flat slabs and the one-way slabs (approximately 1.85 to 2.00). These are recommended provisions for the American Concrete Institute's Building code of 1970[77].

One of the reasons for obtaining a higher factor of safety for the one-way slabs when tested to failure, (referring to NBO tests) as compared with the factor of safety by the working stress design method, is the inelastic stress distribution at failure as compared to the elastic (straight-line theory) stress distribution assumed in the design. Further

the ultimate moment carrying capacity of sections is increased by the presence of strain-hardening, especially for members with low percentage of steel and/or the presence of compression reinforcements Fig. 3.1. In the behaviour of the two-way slabs with the uniformly spaced orthotropic reinforcement, the load carrying capacity is further increased because of the moment redistribution capacity of sections with high ductility (Fig. 3.1) resulting from low percentage of mild steel and/or the use of the compression reinforcement in the form of temperature and shrinkage reinforcement. Failure is reached when sufficient number of points in the slab have yielded to form a collapse mechanism i.e. the plastic moment (ultimate moment capacity) remains constant at points where the yielding first occurs, leading to the redistribution of moments till enough plastic hinges are developed so that a failure mechanism is formed. The presence of the membrane action especially in the case of restrained slabs, slabs supported on beams etc., increases further the load carrying capacity of slabs.

The theoretical factor of safety for flexural members other than the two-way slabs ranges from 1.6 to 1.95 depending upon the usage of the structure. IS:456-1964 specifies a range of 1.72 to 1.95 for the live load/dead load ratio ranging from 0.66 to 2.0 respectively. This is quite a practical range for the design of floors for loading class No. 200, 300, 400 and 1000.[12] (Fig. 3.2).

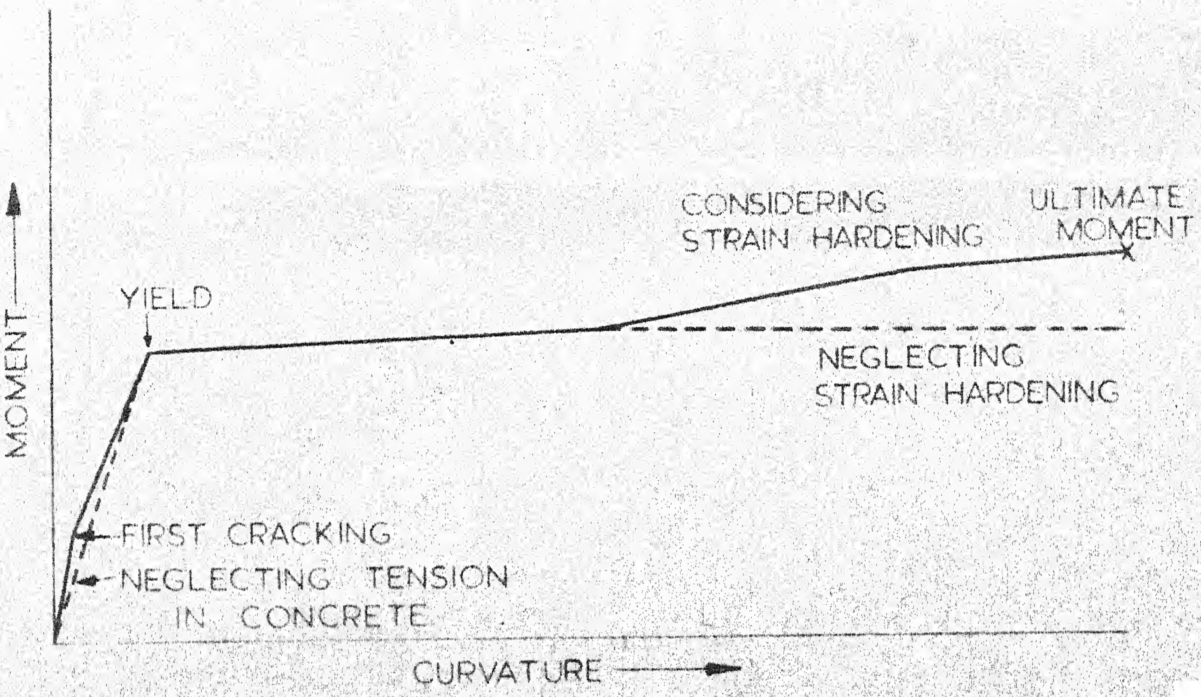


FIG. 3.1 TYPICAL MOMENT CURVATURE CURVE FOR REINFORCED CONCRETE FLEXURAL MEMBER WITH LOW PERCENTAGE OF MILD STEEL REINFORCEMENT.

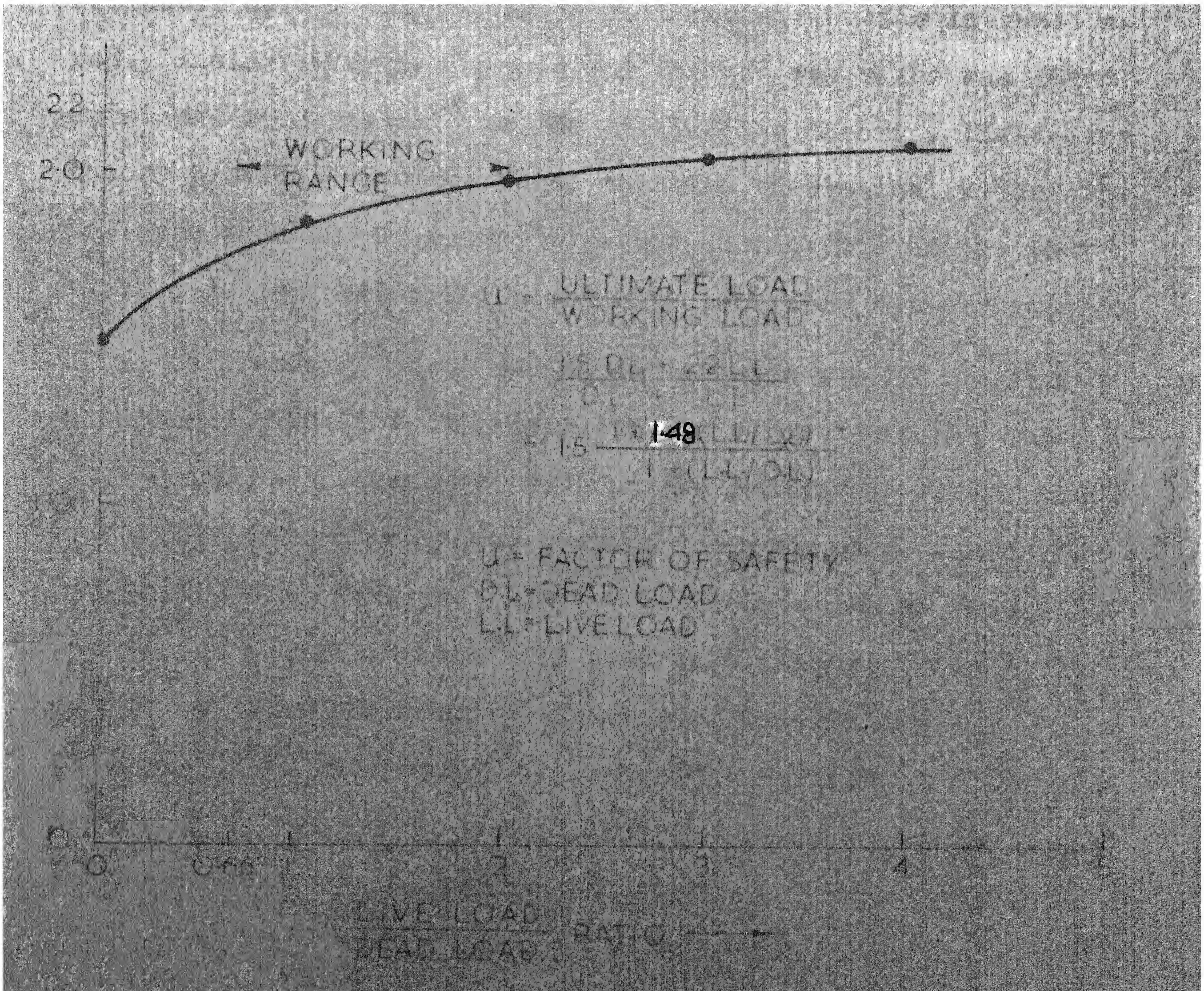


FIG. 3.2 FACTOR OF SAFETY FOR FLEXURAL MEMBERS OTHER THAN THE TWO WAY SLABS FOR DIFFERENT LIVE LOAD/DEAD LOAD RATIO AS PER IS:456-1964[7]

Excessive safety for the two-way slabs designed by the elastic methods specified in various codes of practice, and inherent reserved strength due to the ductile behaviour (Fig.3.1) led to the development of the rational design procedure i.e. inelastic design method, where full redistribution is considered. Inelastic design method for the two-way slabs was first developed by Johansen[23] as the 'yield-line Theory'. Yield-line theory provides an upper bound solution theoretically for the collapse load and is mainly a method of analysis except that it is a design method for slabs with isotropic reinforcements.

Hillerborg[16] proposed an equilibrium theory and strip method, by which the two-way slabs can be designed for the orthotropic reinforcement, without the provision of torsion reinforcement and yet these provide lower bound solutions. So the design will be safe. Similar to the yield-line theory,[23] Hillerborg's Strip Method[10] is also an ultimate strength theory and it will provide distribution of moments for the ultimate load. If this moment distribution is divided by the factor of safety required, moment distribution for working load can be derived and thereafter the design is carried out by WSD. This procedure of 'Inelastic Analysis - Elastic Design' is not new. It is in the use in Scandanavian countries like Sweden etc. and has shown satisfactory performance both at the working loads and the ultimate loads. Thus, it satisfies both, the strength and the serviceability criterias.

Optimal safety as required in Fig. 3.2 is attained indirectly by using Hillerborg's strip method with some modifications founded on valid assumptions, for the uniformly spaced orthotropic reinforcement without the provision of torsion reinforcement, and satisfying the serviceability requirements by restricting the span/thickness ratios and the design by WSD.

3.2 FACTORS OF SAFETY OF TWO-WAY SLABS DESIGNED ACCORDING TO METHOD 2 OF IS: 456-1964.

NBO tests showed that the actual factor of safety of the two-way slab designed according to the IS code of practice[7] is 7.47, while the average for the one-way slabs is 3.5. These factors of safety are very much higher than the computed factors of safety based on the yield-line analysis, due to the effect of strain-hardening and membrane action. The theoretical factor of safety of the two-way slabs with the uniformly spaced orthotropic reinforcement is now investigated, assuming the following relation between the ultimate moment (equal to the yield moment) to the moment at working loads,

$$M_{yield} = K \frac{\sigma_{sy}}{\sigma_{st}} M_{working} \qquad \ldots \quad (3.1)$$

where σ_{sy} = Yield stress. This will have the value of 2600 Kg/cm^2 for Grade I mild steel plain bars,

σ_{st} = Allowable stress in Grade I mild steel bars, in tension. It has the value of 1400 Kg/cm^2.

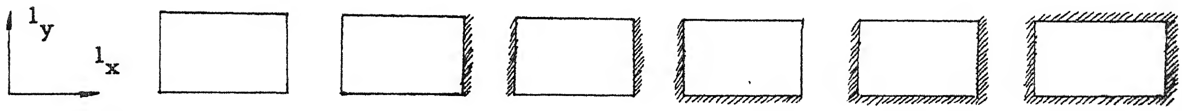
K = Ratio of lever arms at the ultimate load and that at working load. This works out to be approximately 1.05 for the slabs as percentage of steel used is very low, in general.

In above equation (3.1), factor $K \frac{\sigma_{sy}}{\sigma_{st}}$ for IS: 456-1964[7] works out to be 1.95 and this is the factor of safety, as the ultimate load q is proportional to M_{yield} and $q_{working}$ is proportional to $M_{working}$. Thus the value of $K \frac{\sigma_{sy}}{\sigma_{st}}$ satisfies requirements of the factor of safety of IS:456-1964 for the live load/dead load ratio ranging from 0.66 to 2.0.

Table 2 shows values of the theoretical factors of safety for different types of panels and span ratios, for the case, when no torsion reinforcement is provided. These has been found from the collapse load computed considering the collapse of individual panels, using moment coefficients of Method 2 of IS:456-1964[7] and using formula discussed in Appendix D. The actual value of the collapse load will be more in the case of a fixed or continuous boundaries. The range of the theoretical factor of safety is 2.26 to 2.79, while the corresponding value for the one-way slabs and simply supported beams is ranging between 1.76 to 1.95 as has been stated before.

To take the advantages of the plastic reserve strength of the two-way slabs, and to bring down the factor of safety to a rational optimum level and in parity with that of the one-way slabs, the use of Hillerborg's Strip Method[16] with some modifications is made.

TABLE 2: VALUES OF FACTOR OF SAFETY FOR 'MHSM' WITH UNIFORM MOMENT DISTRIBUTION AND NO TORSION REINFORCEMENT AT EXTERNAL CORNERS.



l_y/l_x	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
0.6	1.95 (2.26)	1.99 (2.51)	1.95 (2.47)	1.98 (2.49)	1.97 (2.56)	1.95 (2.54)
0.7	1.95 (2.31)	2.00 (2.56)	1.95 (2.52)	1.98 (2.67)	1.97 (2.67)	1.95 (2.72)
0.8	1.95 (2.31)	1.92 (2.56)	1.95 (2.52)	1.98 (2.67)	1.96 (2.67)	1.95 (2.72)
0.9	1.95 (2.32)	1.92 (2.56)	1.96 (2.53)	1.98 (2.71)	2.00 (2.68)	1.95 (2.77)
1.0	1.95 (2.31)	1.92 (2.55)	1.98 (2.52)	1.98 (2.71)	2.00 (2.66)	1.95 (2.79)
1.1	1.95 (2.31)	1.93 (2.54)	1.99 (2.53)	1.98 (2.71)	2.00 (2.66)	1.95 (2.78)
1.2	1.95 (2.32)	1.95 (2.53)	2.01 (2.52)	1.98 (2.68)	2.01 (2.66)	1.95 (2.75)
1.3	1.96 (2.33)	1.96 (2.53)	2.04 (2.52)	1.99 (2.67)	2.01 (2.63)	1.96 (2.73)
1.4	1.97 (2.34)	1.98 (2.54)	2.06 (2.54)	2.00 (2.65)	2.03 (2.60)	1.97 (2.71)
1.5	1.98 (2.35)	1.99 (2.53)	2.08 (2.55)	2.01 (2.61)	2.04 (2.59)	1.98 (2.68)
1.6	1.99 (2.35)	2.01 (2.53)	2.09 (2.55)	2.03 (2.62)	2.06 (2.59)	1.99 (2.66)

Note: 1. Values in the bracket are for Method 2 of IS: 456-1964
2. A crosshatched edge indicates that the slab continues across or is fixed at the support. An unmarked edge indicate support at which torsional resistance is neglected.

3.3 HILLERBORG'S EQUILIBRIUM THEORY AND STRIP METHOD

Hillerborg proposed an 'Equilibrium Theory' which is simple to apply and which gives results on the safe side. The theory is stated as follows[16]:

"If a distribution of moments can be found which satisfies the plate equilibrium equation and boundary conditions for a given external load, and if the plate at every point, is able to carry these moments, then the given external load represent a lower limit of the carrying capacity of the plate.

The fundamental requirement of the theory is the elasto-plastic behaviour of reinforced concrete slabs with large rotation capacity which is satisfied as the percentage of steel is low, (Fig. 3.1). The theory assumes that the moment is the only significant stress resultant and that premature failures are avoided. Full yield strength of the reinforcing mild steel is assumed to be developed at the critical sections at failure.

The equilibrium equation for plates can be written as:[78]

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q \qquad \ldots \quad (3.2)$$

where M_x. M_y are moments per unit width in X and Y directions respectively in cartesian coordinates,

M_{xy} is the twisting moment per unit width,

q is the load per unit area.

The above equation can be solved by many ways depending upon the nature of the assumptions.[16,42]

Case 1: Assuming $M_{xy} = 0$ and the intensity of load to be distributed equally in both directions, the equations become

$$\frac{\partial^2 M_x}{\partial x^2} = -\frac{q}{2} \qquad \ldots \quad (3.3)$$

$$\frac{\partial^2 M_y}{\partial y^2} = -\frac{q}{2} \qquad \ldots \quad (3.4)$$

Equations (3.3) and (3.4) can be solved with the boundary conditions thereby giving moment fields M_x, M_y for the slab.

Case 2: Assuming all three terms of Equation (3.2) share equal loads, we have

$$\frac{\partial^2 M_x}{\partial x^2} = -\frac{q}{3} \qquad \ldots \quad (3.5)$$

$$\frac{\partial^2 M_y}{\partial y^2} = -\frac{q}{3} \qquad \ldots \quad (3.6)$$

$$\frac{\partial^2 M_{xy}}{\partial x \partial y} = -\frac{q}{6} \qquad \ldots \quad (3.7)$$

Case 3: Here the twisting moments $M_{xy} = 0$ and the load is shared by regions R_x and R_y (Fig. 3.3). In the region R_x the load intensity q is resisted by strips in x direction only and in the region R_y by strips in the y direction only, as is seen from Fig. 3.3. Case 3

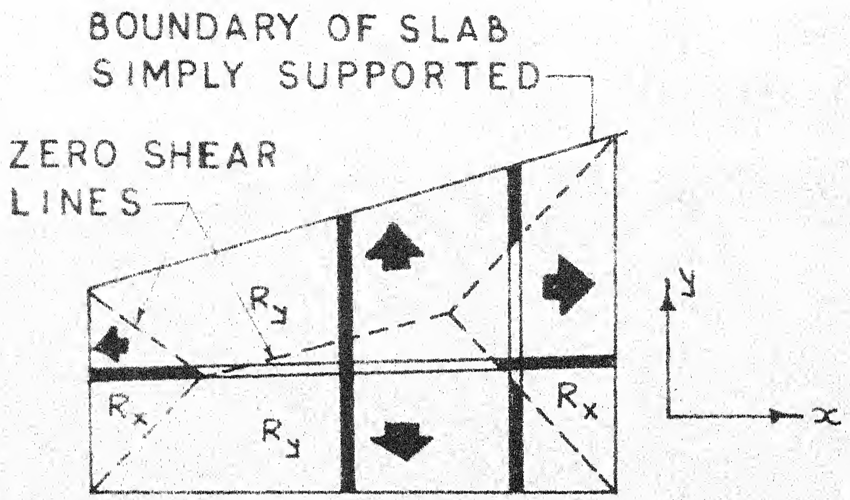


FIG. 3.3 DISTRIBUTION OF LOAD IN REGIONS R_x AND R_y.

is called the Hillerborg's Strip Method.

Equation (3.2) can be rewritten as

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} = - q \qquad \ldots \quad (3.8)$$

In the region R_x, the load distribution is such that

$$\left.\begin{aligned} \frac{\partial^2 M_x}{\partial x^2} &= - q \\ \frac{\partial^2 M_y}{\partial y^2} &= 0 \end{aligned}\right\} \qquad \ldots \quad (3.9)$$

while in the region R_y

$$\left.\begin{aligned} \frac{\partial^2 M_x}{\partial x^2} &= 0 \\ \frac{\partial^2 M_y}{\partial y^2} &= -q \end{aligned}\right\} \qquad \ldots \quad (3.10)$$

If the regions R_x and R_y are known, Equations (3.9), (3.10) yield simple solutions for different boundary conditions (Fig. 3.3). Regions R_x and R_y are bounded by zero shear lines ($M_{xy} = 0$) and the boundaries of the plate. They are similar to the yield lines used in Johansen's theory.[23]

Hillerborg suggests the following general rules for the division of the slab into strips:[16,17]

(1) The strips are laid out so as to carry the load to the

nearest support.

(2) When two simply supported or two fixed edges meet at right angles, for uniform orthotropic reinforcement, the zero shear line can be drawn at 45^{o} from the corner i.e. with a 1:1 slope.

(3) The fixed edge repells the zero shear line while the simply supported edge attracts it. For cases when one support is simply supported while the adjacent support at right angles is fixed, a 2:3 slope is suggested for the zero shear line (Fig. 3.4). (This assumption is strictly true for the case of an isotropically reinforced square slab, with two adjacent edges fixed and the other two simply supported, carrying uniformly distributed load.) Once the R_x, R_y regions are known, the distribution of moments M_x and M_y which satisfy equilibrium theory for reinforced concrete slabs, can be found.

3.3.1 Weakness of Strip Method

The inherent weakness of the strip method is that it offers a very conservative lower bound on the collapse load as shown by Crawford.[42] This shortcoming is overcome by modifying the moment distribution by the principle of weighted averages as suggested by Hillerborg, so as to compute the average moment per unit width of the slab.[17] This is called hereafter modified Hillerborg's Strip Method.

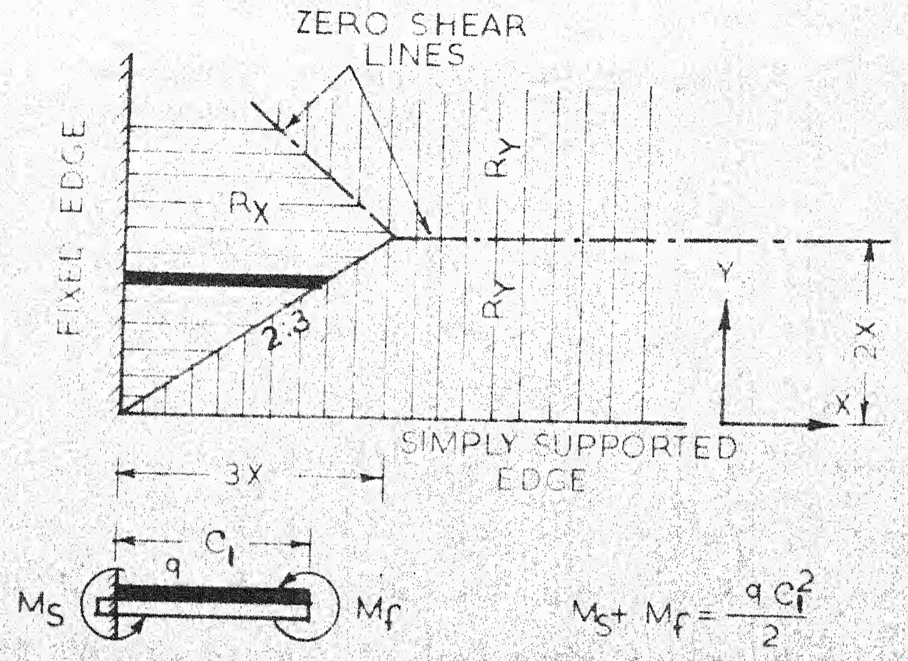


FIG. 3·4 DIVISION OF SLAB INTO STRIPS WITH UNEQUAL BOUNDARY CONDITIONS OF SUPPORTED EDEGS AT CORNER

3.4 MODIFIED HILLERBORG'S STRIP METHOD

3.4.1 Basis and Assumptions

If complete plastic redistribution is available, the moment distribution curve can be averaged to that of an uniform moment called weighted average moment so that equilibrium is satisfied at the ultimate load, but not necessarily the collapse mechanism. In general, because of the large rotation capacity of reinforced concrete slabs met with in the usual applications, the collapse mechanism is always formed. This concept is implicitly used in the design of multi-panel flat slabs wherein the reinforcement is kept constant over strips (column strip and middle strip) of one half of the span length, even though the elastic moments at working loads are quite high than the moment field for an interior panel provided by ACI:318-63[6] (Fig. 3.5). This substantiates the validity of the weighted average process to derive the desired moment distribution. Use of the yield line theory in checking the factor of safety is also made herein to check strength. However the above approach need not guarantee serviceability at working loads. For this it is important to specify the limits on span/thickness ratio so as to control deflections and cracking especially when high strength steels are used. These are specified in the codes of practice.

In this method, therefore, the average moment distribution per unit width of the slab, for uniform orthotropic

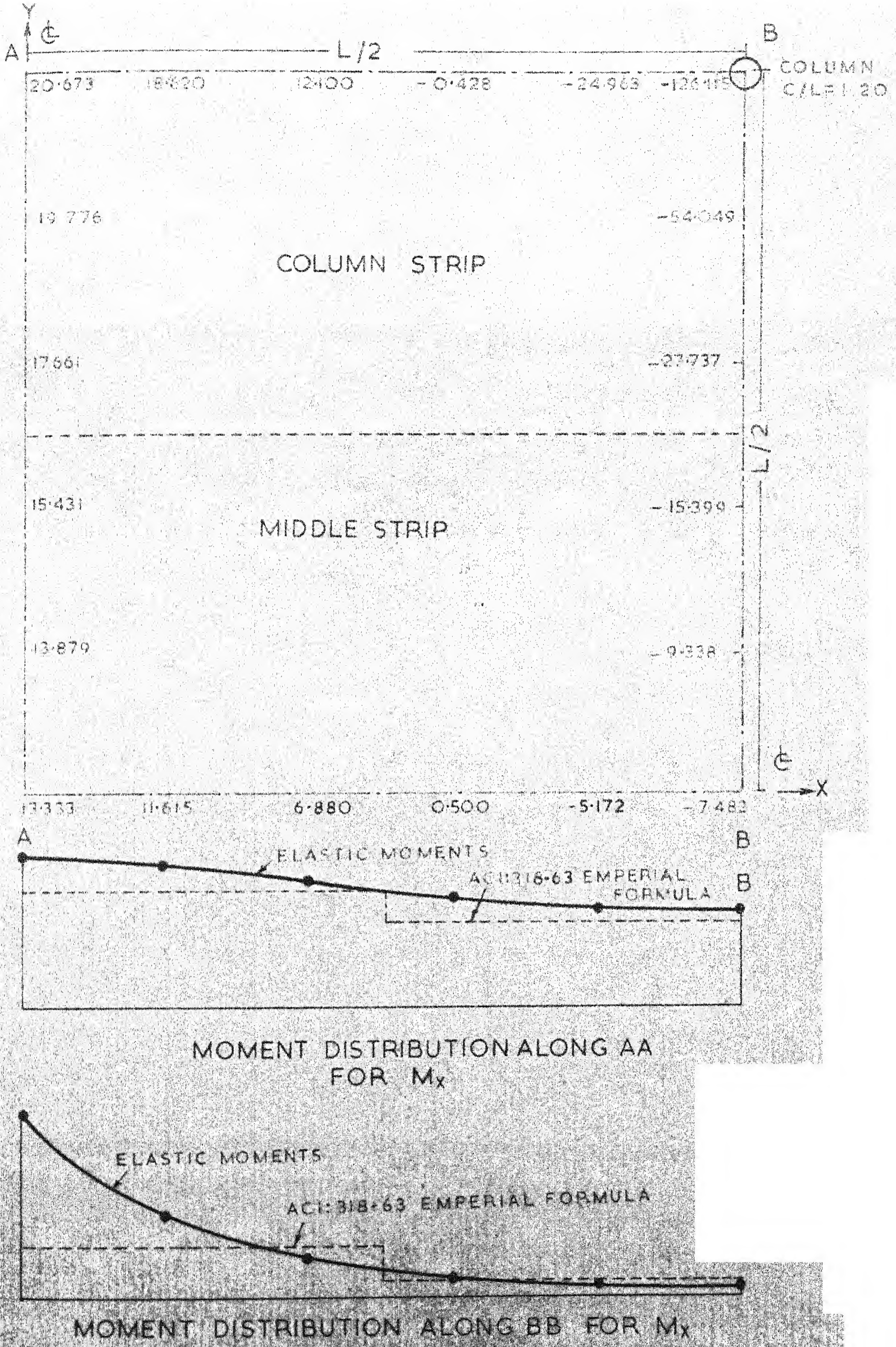


FIG. 3·5 MOMENT DISTRIBUTION IN COLUMN AND MIDDLE STRIP IN X DIRECTION IN THE INTERIOR PANEL OF FLAT SLAB (WITHOUT DROP) BY ELASTIC THEORY (SRIDAAR RAO[79]) & ACI:318-63

reinforcement throughout the whole length of the slab, is derived; as can be seen from Fig. 2.6. For simplicity, the following assumptions are made:

(1) The supporting edges are assumed to be rigid and either perfectly fixed (continuous slabs may be considered in this category for dead loads) or simply supported. However, Hillerborg's strip method can also be used in the design of complex slabs assuming partial fixity of the supports.

(2) The distribution between the span moment (M_f) and the support moment (M_s) is assumed as follows:[17]

$$M_s = 2 M_f \qquad \ldots \qquad (3.11)$$

(This is strictly true for propped cantilevers and fixed beams with uniformly distributed load, for elastic range which is a good index of serviceability behaviour). Applying this to the example (Fig. 3.4). From Equilibrium

$$M_s + M_f = \frac{qc^2_1}{2} \qquad \ldots \qquad (3.12)$$

So,

$$M_s = \frac{qc^2_1}{3} \quad \text{and} \quad M_f = \frac{qc^2_1}{6} \qquad (3.13)$$

(3) When two opposite edges do not have same boundary conditions, then the average span moment M_f is assumed as the maximum of values found for the corresponding regions

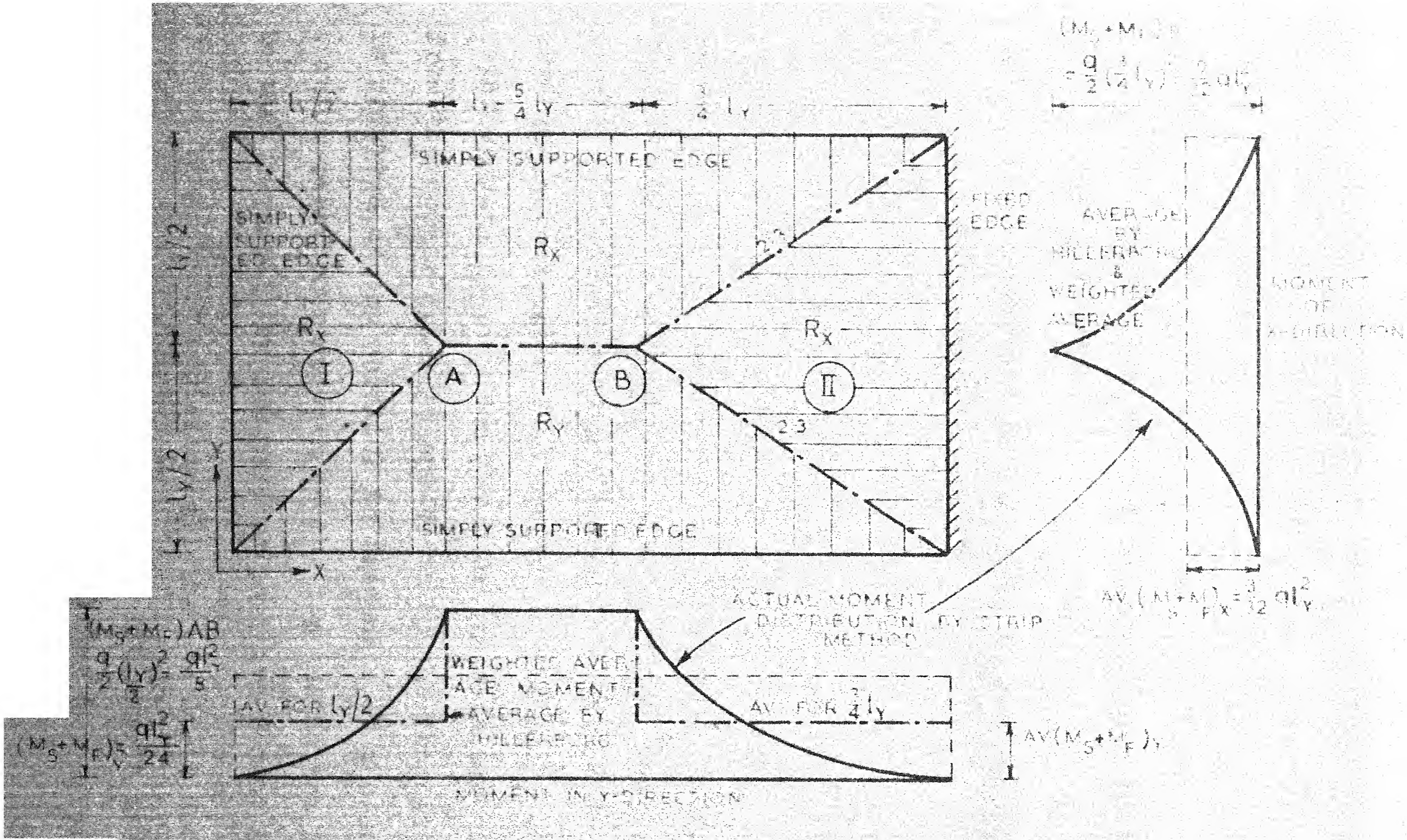


FIG. 3·6 USE OF PRINCIPLE OF WEIGHTED AVERAGES FOR DETERMINING MOMENT FIELD FOR RECTANGULAR PLATE WITH ONE CONTINUOUS EDGE WITH $l_y/l_x \leqslant 0{\cdot}75$

(segments or strips as shown in Fig. 3.6).

(4) When this method is applied to different cases of rectangular slabs for uniformly distributed loads the critical span ratio for each case is calculated, because the pattern of zero shear lines is dependent on the span ratio.

(5) Yield criterion as discussed in section 3.4.5 is approximately satisfied.

3.4.2 Application to Different Cases

Various codes of practice suggest different type of moment distribution across the span (for entire width of slab) as shown in Fig. 3.7. IS: 456-1964[7] Method 2 prescribes moment coefficients with or without the provision of torsion reinforcement. ACI:318-63 requires torsion reinforcement at exterior corners in both the bottom and top of the slab. The torsion reinforcement is provided for a distance in each direction from the corner equal to one-fifth of the longest span and it is provided at top parallel to diagonals at the corner. The torsion reinforcement both at the top and bottom is to be equal to that required for the maximum positive moment in the slab.

Here, using modified Hillerborg's Strip Method moment coefficients have been computed for the moment distributions considered in Fig. 3.6, for case (b) and (a) only.

3.4.2.1 Uniform Moment Distribution Across the Span

Referring Fig. 3.7(b), without the provision of torsion reinforcement, moment coefficients have been computed, similar to those of Method 2 of IS:456-1964[7], firstly for particular case as shown in Fig. 3.6 and then for all cases with different boundary conditions.

To describe the application of modified Hillerborg's strip method to the cases considered in Method 2 of IS:456-1964 a particular case of an uniformly orthotropically reinforced rectangular slab of dimensions l_x, l_y with simply supported conditions on three sides and a fixed edge on the fourth side is shown herein. (the critical span ratio $\frac{l_y}{l_x} = 0.75$) For this case, Fig. 3.6 shows the strips considered for $\frac{l_y}{l_x} \leq 0.75$ and the zero shear lines.

For M_y

The weighted average of $(M_s + M_f)$ in the y direction is given by

$$\frac{1}{l_x} \left[\frac{ql_y^2}{24} \left(\frac{l_y}{2} + \frac{3}{4} l_y \right) + \frac{ql_y^2}{8} \left(l_x - \frac{5}{4} l_y \right) \right] \quad \ldots \quad (3.14)$$

Here $(M_s)_y = 0$, and denoting $\frac{l_y}{l_x} = s$, we obtain

average of (M_f) in the y direction $= M_y$

$$= \frac{ql_y^2}{24} (3 - 2.5s) \quad \ldots \quad (3.15)$$

If this is written in the form $M_y = \frac{ql_y^2}{m_y}$ where m_y is the moment coefficient for positive moment in the y direction,

$$m_y = \frac{24}{(3-2.5s)} \quad \text{for} \quad s \leq 0.75 \qquad \ldots \quad (3.16)$$

For M_x and M_x'

From Fig.3.6 using the principle of weighted averages,

For region I,

$$\text{average} \quad (M_s + M_f)_x = \frac{1}{3} \; \frac{q}{2} \left(\frac{l_y}{2}\right)^2 = \frac{q \; s^2 \; l_x^2}{24} \ldots \quad (3.17)$$

Since $(M_s)_x = 0$, average $(M_f)_x = ql_x^2 \; \frac{s^2}{24}$

For region II,

$$\text{average} \; (M_s + M_f)_x = \frac{3}{32} \; ql_y^2 = \frac{3}{32} \; s^2 \; ql_x^2$$

Using $M_x' = 2M_x$ where $M_x' = (M_s)_x$ = Negative moment at support and $M_x \; (M_f)_x$ = Span moment

We obtain $M_x' = \frac{s^2}{16} \; ql_x^2$

and $M_x = \frac{s^2}{32} \; ql_x^2$ $\qquad \ldots \quad (3.18)$

From regions I and II, the maximum value of M_x for each of the region is taken as M_x for the slab

So, $$M_x = q l_x^2 \frac{s^2}{24} \qquad \ldots (3.19)$$

If $$M_x = \frac{q l_x^2}{m_x}, \text{ then } m_x = \frac{24}{s^2}$$

where m_x = moment coefficient for positive moment in the x direction (same form as in Method 2 of IS:456-1964).

Also m'_x = moment coefficient for negative moment in x direction

$$M'_x = \frac{q l_x^2}{m'_x} \text{ where } m'_x = \frac{16}{s^2} \text{ for } s \leq 0.75.$$

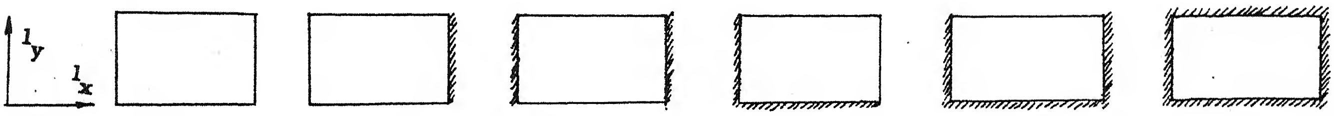
For $s \geq 0.75$ the above procedure is followed after drawing the zero shear lines.

This procedure is applied to six different cases of rectangular slabs with different boundary conditions and the moment coefficients m_x, m'_x, m_y, m'_y for span ratios ranging from 0.6 to 1.6 have been tabulated in Table 3 and 4, using the following formulae.

Case 1 $\quad s \leq 1$

$$\left.\begin{array}{l} m_x = \dfrac{24}{s^2}; \quad m'_x = 0 \\ m_y = \dfrac{24}{(3-2s)}; \quad m'_y = 0 \\ s \geq 1 \\ m_x = \dfrac{24s}{(3s-2)}; \quad m'_x = 0 \\ m_y = 24s^2; \quad m'_y = 0 \end{array}\right\} \qquad \ldots (3.20)$$

TABLE 3: POSITIVE MOMENT COEFFICIENTS FOR DESIGN OF TWO-WAY R.C. SLAB WITHOUT PROVISION OF TORSION REINFORCEMENTS AT DISCONTINUOUS EDGES

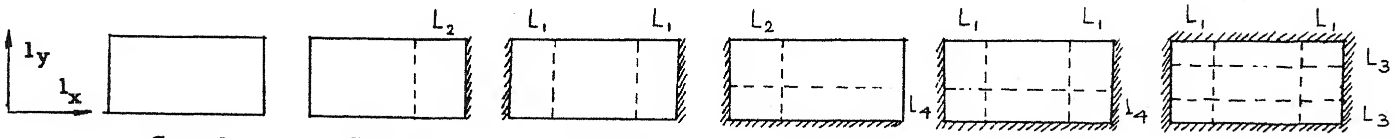


	Case 1		Case 2		Case 3		Case 4		Case 5		Case 6	
l_y/l_x	m_x	m_y	m_x	m_y	m_x	m_y	m_x	m_y	m_x	m_y	m_x	m_y
0.6	(66.5	13.3	66.5	16.0	89.0	20.0	119.	23.7	128.	28.5	199.	39.9
	(80.5)	(10.4)	(69.0)	(11.9)	(87.6)	(16.1)	(133.)	(17.4)	(138.)	(20.7)	(229.)	(29.8)
0.7	49.0	15.0	49.0	19.2	65.6	26.6	87.1	26.7	94.0	34.2	147.	45.0
	(49.5)	(11.9)	(46.3)	(14.7)	(63.7)	(21.8)	(80.7)	(19.5)	(90.7)	(24.9)	(139.)	(33.4)
0.8	37.5	17.2	41.2	27.4	54.0	34.0	66.7	30.5	71.9	42.8	112.	51.6
	(33.9)	(13.9)	(34.6)	(18.7)	(50.4)	(29.6)	(54.6)	(22.4)	(66.3)	(31.0)	(94.5)	(38.8)
0.9	29.6	20.0	34.5	34.5	47.3	34.8	52.7	35.6	62.7	43.8	88.8	60.0
	(25.4)	(16.6)	(27.9)	(24.2)	(42.5)	(40.8)	(40.5)	(26.6)	(52.5)	(39.4)	(70.1)	(46.0)
1.0	24.0	24.0	30.7	42.7	43.1	54.0	42.7	42.7	54.0	54.0	72.0	72.0
	(20.0)	(20.2)	(23.9)	(31.8)	(37.5)	(55.7)	(32.2)	(32.2)	(44.2)	(50.6)	(55.7)	(55.7)
1.1	20.3	29.0	28.1	51.8	40.3	64.5	36.1	51.7	48.4	65.2	60.9	87.0
	(16.9)	(24.8)	(21.4)	(42.0)	(34.2)	(75.5)	(27.0)	(39.6)	(38.9)	(65.2)	(46.8)	(68.5)
1.2	18.0	34.6	28.3	61.5	38.0	78.0	32.0	62.2	44.8	77.8	54.0	104.
	(14.7)	(30.6)	(19.6)	(55.0)	(31.9)	(102.)	(23.7)	(49.2)	(35.3)	(84.2)	(40.9)	(85.0)
1.3	16.4	40.6	24.9	72.1	36.4	91.5	29.2	72.1	42.0	91.2	49.2	122.
	(13.2)	(37.6)	(18.4)	(71.0)	(30.3)	(135.)	(21.4)	(61.0)	(32.8)	(108.)	(36.9)	(105.)
1.4	15.5	47.0	23.9	85.0	35.2	106.	27.2	83.5	39.8	106.	45.9	141.
	(12.1)	(46.6)	(17.6)	(91.5)	(29.2)	(176.)	(19.8)	(76.0)	(31.0)	(138.)	(34.1)	(131.)
1.5	14.4	54.0	23.0	96.0	34.1	121.	25.6	96.0	38.2	122.	43.2	162.
	(11.3)	(57.5)	(17.0)	(118.)	(28.3)	(227.)	(18.7)	(95.0)	(29.7)	(175.)	(32.0)	(162.)
1.6	13.7	61.5	22.3	109.	33.2	167.	24.4	109.	37.2	137.	41.1	185.
	(10.8)	(70.6)	(16.5)	(148.)	(27.6)	(288.)	(17.8)	(117.)	(28.7)	(219.)	(30.6)	(200.)

Note: 1. **Values in the bracket are from Table XIV of IS: 456-1964.**

2. **A crosshatched edge indicates that the slab continues across or is fixed at the support, an unmarked edge indicates support at which torsional resistance is neglected.**

TABLE 4: NEGATIVE (SUPPORT) MOMENT COEFFICIENTS FOR DESIGN OF TWO-WAY R. C. SLAB WITHOUT PROVISION OF TORSION REINFORCEMENTS AT DISCONTINUOUS EDGES



Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6

l_y/l_x	Case 1 m'_x	Case 1 m'_y	Case 2 m'_x	Case 2 m'_y	Case 3 m'_x	Case 3 m'_y	Case 4 m'_x	Case 4 m'_y	Case 5 m'_x	Case 5 m'_y	Case 6 m'_x	Case 6 m'_y
0.6			43.5 (32.6)		43.5 (30.6)		63.9 (69.1)	12.8 (09.0)	64.0 (58.0)	15.3 (10.1)	100. (104.)	20.0 (13.4)
0.7			32.6 (21.3)		32.8 (22.0)		47.0 (41.2)	14.4 (09.9)	47.0 (37.0)	18.4 (11.8)	73.5 (61.6)	22.5 (14.9)
0.8			20.5 (15.7)		27.0 (17.8)		36.0 (27.4)	16.4 (11.3)	35.9 (26.6)	23.0 (14.6)	56.2 (41.2)	25.7 (17.0)
0.9			17.2 (12.9)		23.7 (15.6)		28.4 (20.2)	19.2 (13.3)	31.4 (21.1)	29.2 (18.5)	44.5 (30.3)	30.0 (19.9)
1.0			15.3 (11.2)		21.6 (14.4)		23.0 (16.0)	23.0 (16.0)	27.0 (18.0)	36.0 (24.0)	36.0 (24.0)	36.0 (24.0)
1.1			14.0 (10.2)		20.1 (13.6)		19.4 (13.5)	27.8 (19.7)	24.2 (16.1)	43.2 (31.1)	30.5 (20.2)	43.5 (29.5)
1.2			13.2 (09.6)		19.0 (13.2)		17.3 (11.9)	33.2 (24.6)	22.3 (14.6)	52.1 (41.3)	27.0 (17.8)	51.8 (37.0)
1.3			12.5 (09.1)		18.2 (12.9)		15.7 (10.8)	38.9 (31.0)	21.0 (14.1)	61.1 (53.9)	24.6 (16.2)	60.8 (46.4)
1.4			12.0 (08.8)		17.6 (12.6)		14.6 (10.1)	45.0 (38.8)	19.9 (13.6)	70.8 (96.5)	22.9 (15.1)	70.5 (58.3)
1.5			11.5 (08.6)		17.1 (12.4)		13.8 (09.6)	51.7 (51.5)	19.1 (13.2)	81.5 (89.0)	21.6 (14.4)	81.0 (72.6)
1.6			11.1 (08.5)		16.6 (12.3)		13.1 (09.2)	59.0 (61.6)	18.4 (12.9)	92.6 (112.)	20.6 (13.8)	92.2 (91.0)

Note: 1. Values in the bracket are from Table XIV of IS: 456-1964.

2. A crosshatched edge indicates that the slab continues across or is fixed at the support, an unmarked edge indicates support at which torsional resistance is neglected.

Case 2 $\quad s \leq 0.75$

$$m_x = \frac{24}{s^2} \; ; \; m_x' = \frac{16}{s^2}$$

$$m_y = \frac{24}{(3-2.50s)} \; ; \; m_y' = 0$$

$s \geq 0.75$

$$m_x = \frac{30.7s}{(2s-1)} \; ; \; m_x' = \frac{15.35s}{(2s-1)}$$

$$m_y = 42.7\, s^2 \; ; \; m_y' = 0$$

$$\ldots (3.21)$$

Case 3 $\quad s \leq 0.667$

$$m_x = \frac{32}{s^2} \; ; \; m_x' = \frac{16}{s^2}$$

$$m_y = \frac{8}{1-s} \; ; \; m_y' = 0$$

$s \geq 0.667$

$$m_x = \frac{108s}{(4.5s-2)} \; ; \; m_x' = \frac{27s}{(2.25s-1)}$$

$$m_y = 54\, s^2 \; ; \; m_y' = 0$$

$$\ldots (3.22)$$

<u>Case 2</u>

$$\left.\begin{aligned}
&s \leq 0.75\\
&\qquad m_x = \frac{24}{s^2} ; \quad m'_x = \frac{16}{s^2}\\
&\qquad m_y = \frac{24}{(3-2.50s)} ; \quad m'_y = 0\\
&s \geq 0.75\\
&\qquad m_x = \frac{30.7s}{(2s-1)} ; \quad m'_x = \frac{15.35s}{(2s-1)}\\
&\qquad m_y = 42.7\ s^2 ; \quad m'_y = 0
\end{aligned}\right\} \ldots \quad (3.21)$$

<u>Case 3</u>

$$\left.\begin{aligned}
&s \leq 0.667\\
&\qquad m_x = \frac{32}{s^2} ; \quad m'_x = \frac{16}{s^2}\\
&\qquad m_y = \frac{8}{1-s} ; \quad m'_y = 0\\
&s \geq 0.667\\
&\qquad m_x = \frac{108s}{(4.5s-2)} ; \quad m'_x = \frac{27s}{(2.25s-1)}\\
&\qquad m_y = 54\ s^2 ; \quad m'_y = 0
\end{aligned}\right\} \ldots \quad (3.22)$$

Case 4 $\quad s \leq 1$

$$
\left.
\begin{aligned}
m_x &= \frac{42.67}{s^2} \; ; \quad m'_x = \frac{23}{s^2} \\
m_y &= \frac{42.67}{(3-2s)} \; ; \quad m'_y = \frac{23}{(3-2s)} \\
&s \geq 1 \\
m_x &= \frac{42.67s}{(3s-2)} \; ; \quad m'_x = \frac{23s}{(3s-2)} \\
m_y &= 42.67\, s^2 ; \quad m'_y = 23\, s^2
\end{aligned}
\right\} \quad \ldots \quad (3.23)
$$

Case 5: $\quad s \leq 0.8$

$$
\left.
\begin{aligned}
m_x &= \frac{46.0}{s^2} \; ; \quad m'_x = \frac{23.0}{s^2} \\
m_y &= \frac{17.1}{(1.2-s)} \; ; \quad m'_y = \frac{9.2}{(1.2-s)}
\end{aligned}
\right\} \quad \ldots \quad (3.24)
$$

$s \geq 0.8$

$$\left.\begin{aligned} m_x &= \frac{43.2s}{(1.8s-1)} \;;\quad m'_x = \frac{21.6s}{(1.8s-1)} \\ m_y &= 54s^2 \;;\quad m'_y = 36s^2 \end{aligned}\right\} \quad \ldots (3.24)$$

Case 6: $s \leq 1$

$$\left.\begin{aligned} m_x &= \frac{72}{s^2} \;;\quad m'_x = \frac{36}{s^2} \\ m_y &= \frac{72}{(3-2s)} \;;\quad m'_y = \frac{36}{(3-2s)} \\ &s \geq 1 \\ m_x &= \frac{72s}{(3s-2)} \;;\quad m'_x = \frac{36s}{(3s-2)} \\ m_y &= 72\ s^2 \;;\quad m'_y = 36\ s^2 \end{aligned}\right\} \quad \ldots (3.25)$$

3.4.2 Trapezoidal Moment Distribution Across the Span:

ACI:318-63[6] Method 2 and 3, uses traperoidal moment distribution across the span (as shown in Fig. 3.7(a)) along-with the torsion reinforcement at the external corners.

Here, in this section, use of modified Hillerborg's Strip Method is made to derive the moment coefficients similar to those of Method 3 of ACI: 318-63[6], with reasonable and valid assumption.

Moment carrying capacity at top at corner is resolved, assuming their effectiveness, in x and y directions as shown in Fig. 3.8(i) and (ii). With this assumption, only change to be made is in the internal work done by resisting moments and it can be written as: (Fig. 3.8 (iii)).

$$\left| \frac{5}{6} M_y l_x (1+k_1+k_3) + \frac{5}{6} M_x l_y (1+k_2+k_4) + 8 x 0.707 \text{ x} \right.$$
$$\left. + 8 \text{ x } 0.707 \text{ x } M_y \text{ x } \frac{l_x}{5} \right| \theta = \text{External work done} \quad \ldots(3.26)$$

Entire procedure is shown clearly below with an illustrative example of simply supported case ($s \leq 1$) with an U.D.L. (q).

Illustrative Example

$$k_1 = k_2 = k_3 = k_4 = 0$$

It is worth noting that,

$$N_{ij} = 0 \text{ if } k_i \text{ or } k_j \text{ is non-zero}$$
$$\text{and} \quad N_{ij} = 0 \text{ if } k_i \text{ and } k_j \text{ are zero} \quad \ldots(3.27)$$
$$i \text{ and } j = 1, 2, 3, 4$$

Zero shear lines will be 45° at corners, as

$$k_1 = k_2 = k_3 = k_4 = 0$$

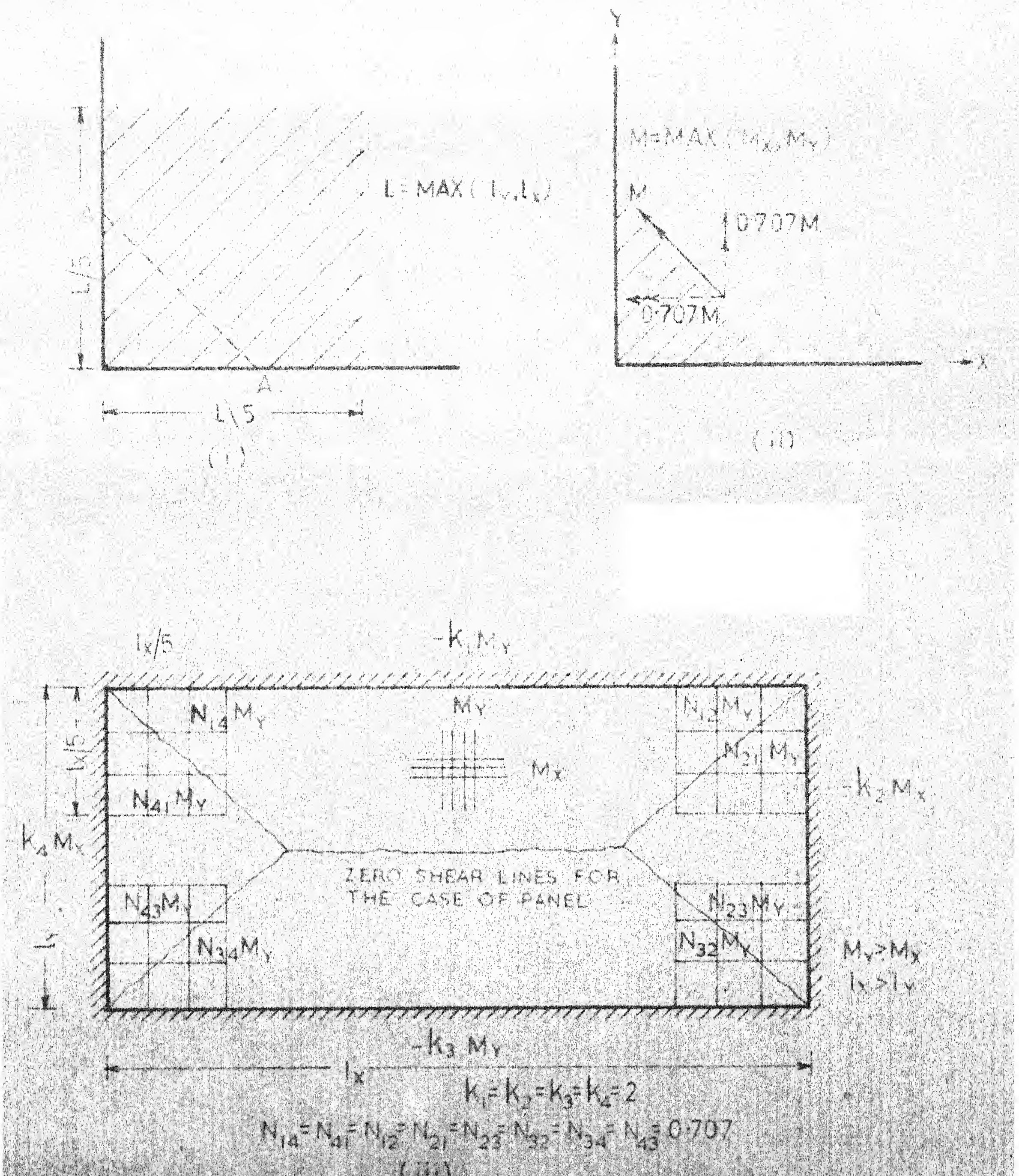


FIG. 3·8 USE OF MODIFIED HILLERBORG'S STRIP METHOD WHEN TORSION REINFORCEMENT IS PLACED DIAGONALLY AT EXTERNAL CORNER (ACI 318-63)[6]

Then, from study of equilibrium of part (1)

$$\frac{5}{6} M_y l_x + 2 \times 0.707 \times M_y \times \frac{l_x}{5}$$

$$= \frac{1}{3} \frac{q}{2} \left(\frac{l_y}{2}\right)^2 \times 2 \times \frac{l_y}{2} + \frac{q}{2} \left(\frac{l_y}{2}\right)^2 \times (l_x - l_y) \quad ..(3.28)$$

and, then

$$M_y = \frac{q l_y^2}{26.7} (3-2s) \quad ... \quad (3.29)$$

Now, from study of equilibrium of part (2)

$$\frac{5}{6} M_x l_y + 2 \times 0.707 \times M_y \times \frac{l_x}{5} = \frac{q l_y^3}{24} \quad ... \quad (3.30)$$

Using Equation (3.29) in above and simplifying,

$$M_x = \frac{q l_x^2 s}{20} (1.5 s - 0.75) \quad ... \quad (3.31)$$

Denoting

$$M_x = C_x q l_x^2$$

$$M_y = C_y q l_y^2$$

$$M'_x = C_{Nx} q l_x^2 \quad ... \quad (3.32)$$

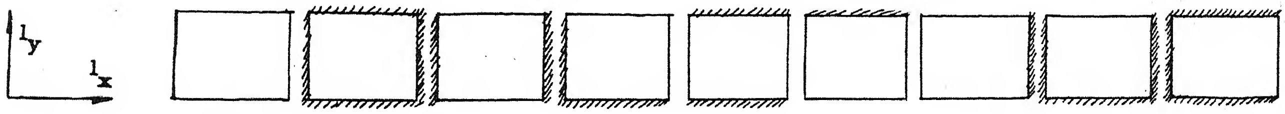
and $$M'_y = C_{Ny} q l_y^2$$

in a form quite similar to Method 3 of ACI:318.63.

Tables 5 and 6 give the moment coefficients for any live load/dead load ratio, for the case of traperoidal moment distribution and the provision of torsion reinforcement at external corners, from the following formulae which are

TABLE 5: POSITIVE MOMENT COEFFICIENTS FOR DESIGN OF TWO-WAY R. C. SLABS WITH TRAPEZOIDAL MOMENT DISTRIBUTION AND TORSION REINFORCEMENT AT EXTERNAL CORNERS.

$M_x = C_x q l_x^2$, $M_y = C_y q l_y^2$ where q = total uniform design load (dead load + live load)

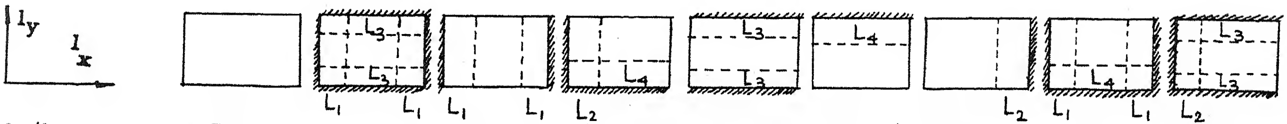


l_y/l_x		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	C_y	0.037	0.017	0.022	0.026	0.028	0.032	0.024	0.022	0.022
	C_x	0.037	0.017	0.028	0.026	0.022	0.023	0.035	0.019	0.024
0.95	C_y	0.041	0.018	0.025	0.029	0.029	0.033	0.027	0.025	0.024
	C_x	0.032	0.015	0.027	0.024	0.020	0.020	0.032	0.017	0.020
0.90	C_y	0.045	0.020	0.027	0.031	0.030	0.035	0.030	0.027	0.025
	C_x	0.027	0.013	0.025	0.021	0.018	0.018	0.029	0.016	0.018
0.85	C_y	0.049	0.022	0.031	0.034	0.031	0.036	0.033	0.031	0.026
	C_x	0.022	0.012	0.024	0.019	0.016	0.015	0.026	0.014	0.016
0.80	C_y	0.052	0.023	0.035	0.037	0.032	0.038	0.038	0.032	0.028
	C_x	0.018	0.011	0.022	0.017	0.014	0.013	0.022	0.013	0.014
0.75	C_y	0.056	0.025	0.040	0.039	0.033	0.039	0.043	0.035	0.029
	C_x	0.014	0.009	0.020	0.015	0.012	0.011	0.018	0.013	0.012
0.70	C_y	0.060	0.027	0.045	0.042	0.034	0.041	0.054	0.035	0.031
	C_x	0.010	0.008	0.018	0.013	0.011	0.009	0.018	0.012	0.011
0.65	C_y	0.064	0.028	0.053	0.044	0.036	0.043	0.059	0.039	0.032
	C_x	0.007	0.007	0.016	0.011	0.009	0.007	0.016	0.011	0.009
0.60	C_y	0.067	0.030	0.060	0.047	0.037	0.044	0.064	0.042	0.033
	C_x	0.004	0.006	0.014	0.009	0.008	0.006	0.013	0.009	0.008
0.55	C_y	0.071	0.032	0.068	0.050	0.038	0.046	0.070	0.046	0.035
	C_x	0.002	0.005	0.011	0.008	0.007	0.004	0.011	0.008	0.007
0.50	C_y	0.077	0.033	0.075	0.052	0.039	0.047	0.075	0.049	0.036
	C_x	0.000	0.004	0.009	0.007	0.006	0.003	0.009	0.007	0.006

Note: 1. A crosshatched edge indicates that the slab continues across or is fixed at the support, an unmarked edge indicates a support at which torsional resistance is negligible.

TABLE 6: NEGATIVE (SUPPORT) MOMENT COEFFICIENTS FOR DESIGN OF TWO-WAY R.C. SLABS WITH TRAPEZOIDAL MOMENT DISTRIBUTION AND TORSION REINFORCEMENT AT EXTERNAL CORNERS.

$M'_x = C_{Nx}\, q\, l_x^2$, $M'_y = C_{Ny}\, q\, l_y^2$ where q = total uniform design load (dead load + live load)



l_y/l_x		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	C_{Ny}		0.033		0.052	0.056	0.078		0.033	0.044
	C_{Nx}		0.033	0.056	0.052			0.078	0.037	0.033
0.95	C_{Ny}		0.037		0.057	0.058	0.082		0.037	0.047
	C_{Nx}		0.030	0.053	0.047			0.074	0.035	0.030
0.90	C_{Ny}		0.040		0.063	0.060	0.086		0.041	0.050
	C_{Nx}		0.027	0.051	0.042			0.069	0.032	0.027
0.85	C_{Ny}		0.043		0.068	0.062	0.090		0.046	0.053
	C_{Nx}		0.024	0.048	0.038			0.064	0.029	0.024
0.80	C_{Ny}		0.047		0.073	0.064	0.094		0.052	0.056
	C_{Nx}		0.021	0.044	0.033			0.059	0.025	0.021
0.75	C_{Ny}		0.050		0.078	0.067	0.098		0.053	0.058
	C_{Nx}		0.019	0.041	0.029			0.052	0.029	0.019
0.70	C_{Ny}		0.053		0.084	0.069	0.102		0.054	0.061
	C_{Nx}		0.016	0.037	0.026			0.037	0.026	0.016
0.65	C_{Ny}		0.057		0.089	0.071	0.105		0.060	0.064
	C_{Nx}		0.014	0.032	0.022			0.032	0.022	0.014
0.60	C_{Ny}		0.060		0.094	0.073	0.109		0.065	0.067
	C_{Nx}		0.012	0.027	0.019			0.027	0.019	0.012
0.55	C_{Ny}		0.063		0.099	0.076	0.113		0.071	0.069
	C_{Nx}		0.010	0.023	0.016			0.023	0.016	0.010
0.50	C_{Ny}		0.067		0.104	0.078	0.117		0.076	0.072
	C_{Nx}		0.008	0.019	0.013			0.019	0.013	0.008

Note: 1. A crosshatched edge indicates that the slab continues across or is fixed at the support, an unmarked edge indicates a support at which torsional resistance is negligible.

derived using Equation (3.26) and the modified Hillerborg's Strip Method.

Case 1: $s \leq 1.0$

$$\left.\begin{aligned} C_x &= \frac{s(1.5\,s - 0.75)}{20} \\ C_y &= \frac{(3-2s)}{26.7} \\ C_{Nx} &= 0 \\ C_{Ny} &= 00 \end{aligned}\right\} \qquad \ldots(3.33)$$

Case 2: $s \leq 1.0$

$$\left.\begin{aligned} C_x &= = \frac{s^2}{60} \\ C_y &= \frac{(3-2s)}{60} \\ C_{Nx} &= \frac{s^2}{30} \\ C_{Ny} &= \frac{(3-2s)}{30} \end{aligned}\right\} \qquad \ldots \quad (3.34)$$

Case 3: $s \leq 1.00$

≥ 0.666

$$\left.\begin{aligned} C_x &= \frac{(4.5\,s - 2)}{90\,s} \\ C_y &= \frac{1}{45\,s^2} \\ C_{Nx} &= \frac{(4.5\,s - 2)}{45\,s} \\ C_{Ny} &= 0 \end{aligned}\right\} \qquad \ldots \quad (3.35)$$

$$s \leq 0.666$$

$$\left.\begin{aligned} C_x &= \frac{s^2}{26.66} \\ C_y &= \frac{(1-s)}{6.66} \\ C_{Nx} &= \frac{s^2}{13.13} \\ C_{Ny} &= 0 \end{aligned}\right\} \quad \ldots \quad (3.36)$$

Case 4: $s \leq 1$

$$\left.\begin{aligned} C_x &= \frac{s^2}{38.3} \\ C_y &= \frac{(3-2\ s)}{38.3} \\ C_{Nx} &= \frac{s^2}{19.15} \\ C_{Ny} &= \frac{(3-2s)}{19.15} \end{aligned}\right\} \quad \ldots \quad (3.37)$$

Case 5: $s \leq 1$

$$\left.\begin{aligned} C_x &= \frac{s^2}{45} \\ C_y &= \frac{(9-4\ s)}{180} \\ C_{Nx} &= 0 \\ C_{Ny} &= \frac{(9-4\ s)}{90} \end{aligned}\right\} \quad \ldots \quad (3.38)$$

Case 6: $s \leq 1$

$$C_x = \frac{s(3.14\ s-1)}{93.7}$$

Case 6: $s \leq 1$

$$\left.\begin{aligned}
C_x &= \frac{s(3.14\ s - 1)}{93.7} \\
C_y &= \frac{(2-s)}{31.7} \\
C_{Nx} &= 0 \\
C_{Ny} &= \frac{(2-s)}{12.8}
\end{aligned}\right\} \quad \ldots \quad (3.39)$$

Case 7: $s \leq 1$

≥ 0.75

$$\left.\begin{aligned}
C_x &= \frac{(1.68\ s - 1)}{19.65\ s} \\
C_y &= \frac{1}{41.4\ s^2} \\
C_{Nx} &= \frac{(2\ s - 1)}{12.8\ s} \\
C_{Ny} &= 0
\end{aligned}\right\} \quad \ldots \quad (3.40)$$

$s \leq 0.75$

$$\left.\begin{aligned}
C_x &= \frac{s^2}{26.7} \\
C_y &= \frac{(3-2.5\ s)}{23.3} \\
C_{Nx} &= \frac{s^2}{13.35} \\
C_{Ny} &= 0
\end{aligned}\right\} \quad \ldots \quad (3.41)$$

Case 8: $s \leq 1.0$

≥ 0.8

$$\left.\begin{aligned} C_x &= \frac{(1.8\ s - 1)}{43.2\ s} \\ C_y &= \frac{1}{45\ s^2} \\ C_{Nx} &= \frac{(1.8\ s - 1)}{21.6\ s} \\ C_{Ny} &= \frac{1}{30\ s^2} \end{aligned}\right\} \qquad \ldots \quad (3.42)$$

$s \leq 0.8$

$$\left.\begin{aligned} C_x &= \frac{s^2}{38.4} \\ C_y &= \frac{(1.2 - s)}{14.25} \\ C_{Nx} &= \frac{s^2}{19.2} \\ C_{Ny} &= \frac{(1.2 - s)}{9.2} \end{aligned}\right\} \qquad \ldots \quad (3.43)$$

Case 9: $s \leq 1$

$$\left.\begin{aligned} C_x &= \frac{s^2}{45} \\ C_y &= \frac{(1.8 - s)}{36} \\ C_{Nx} &= \frac{s^2}{30} \\ C_{Ny} &= \frac{(1.8 - s)}{18} \end{aligned}\right\} \qquad \ldots (3.44)$$

3.4.3 Comparison of MHSM with Codes

3.4.3.1 Comparison with Method 2 of IS: 456-1964:

Tables 3 and 4 give coefficients for the yield (or ultimate) moments for ultimate load intensity q_u. If it is intended that the factor of safety be equal to the ratio of σ_{sy} to σ_{st} for strength and serviceability considerations (this includes use of high strength deformed bars also), and since the assumption made in Equation (3.1) is reasonably true and err on the safe side, the moment coefficients obtained by the modified Hillerborg's strip method can be used for working load intensity q_w and elastic design stresses. These coefficients which take into account the beneficial effects of plasticity of slabs can be compared to the coefficients obtained by elastic analysis of slabs, which consider slabs as two sets of mutually perpendicular strips such that the intensity of load is carried between the strips in such proportions that the deflection at the centre of the slab is same for both the middle strips; as in done in Method 2 of IS: 456-1964[7]. The elastic method also takes into account the correction for torsion and corner restraint. Apparently, no advantage of the ductile behaviour of slabs is taken in the design, thereby leading to high factor of safety as compared to that of one-way slabs.[74] The moment coefficients derived here reduces this discrepancy and an economical design of slabs can be done by the use of the given coefficients. In Tables 3 and 4, along with these values is

given the moment coefficients of Method 2 of IS:456-1964[7] for the cases of rectangular slabs without the use of special torsion reinforcements at corners when the edges are discontinuous. It is seen that the moment coefficients given in the Indian Standard code of practice is conservative and for an economical design use of the values given here is recommended alongwith limitations on span thickness ratios given in the code.

The maximum difference in the bending moments between the proposed coefficients and the coefficients given in Method 2 are of the order of 30 percent especially for negative moments, while on an average the reduction in bending moment is about 20 percent. The reduced coefficients effect economy considerably for span ratios between 0.7 and 1.4 which are practical span ratios for two-way slabs.

For a conservative revision of the Indian Standard code of practice, before full scale tests of slabs designed according to this method is made, the present moment coefficients may be changed in the direction towards the proposed coefficients so that atleast a partial advantage of ductile behaviour of slabs can be utilised in design.

3.4.3.2 Comparison with ACI: 318-63 Method 3

Tables 5 and 6 give moment coefficients in some format as of Method 3 of ACI: 318-63,[6] for positive and negative design moments for trapezoidal moment distribution across the span alongwith the provision of torsion reinforcement at the

external corners, for an ultimate load. From the discussion in the previous section, these moment coefficients can be used for the working load, such that the factor of safety is guaranteed to be atleast σ_{sy}/σ_{st} for strength and serviceability considerations.

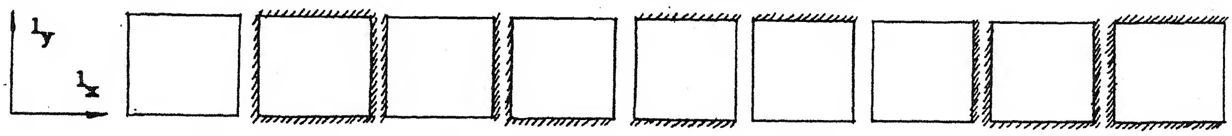
It is evident from Table 7, that Method 3 of ACI: 318-63[6] do not take full advantage of the ductile behaviour (Fig. 3.1) in the design, and thus it is leading to higher factor of safety as compared to MHSM.

Method 3 of ACI: 318-63[6], employs three tables for the design of slabs, for positive moments under dead load, positive moments under live load and negative moments under dead plus live load, leading to complicated, cumbersome and conservative design process. While coefficients presented here are only in two tables, one for the positive moments and the other for the negative moments, assuring serviceability at working loads as discussed in the following section 3.4.4.Tables 5 and 6 are valid for any live load / dead load ratios less than or equal to two.

When the coefficients given here in Tables 5 and 6 are used for the design purposes by WSD, the limitations specified in ACI: 318-63[6] for the span / thickness ratios for panels with the different boundary conditions, are to be satisfied.

Depending upon the live load / dead load ratio, the maximum difference in the bending moments as computed from

TABLE 7: VALUES OF FACTOR OF SAFETY FOR 'MHSM' WITH TRAPEZOIDAL MOMENT DISTRIBUTION AND TORSION REINFORCEMENT AT EXTERNAL CORNERS.



l_y/l_x	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	1.93 (1.86)	1.92 (2.60)	2.25 (2.35)	2.24 (2.11)	2.21 (2.28)	2.07 (2.03)	1.99 (2.01)	1.92 (2.39)	2.27 (2.38)
0.95	1.93 (1.91)	1.92 (2.62)	2.27 (2.31)	2.26 (2.11)	2.22 (2.28)	2.07 (2.07)	1.98 (2.04)	1.94 (2.35)	2.28 (2.36)
0.90	1.93 (1.98)	1.92 (2.64)	2.29 (2.34)	2.29 (2.13)	2.22 (2.26)	2.08 (2.10)	1.98 (2.04)	1.97 (2.35)	2.29 (2.37)
0.85	1.93 (2.05)	1.92 (2.69)	2.31 (2.31)	2.31 (2.14)	2.23 (2.28)	2.08 (2.12)	1.98 (2.07)	2.00 (2.34)	2.30 (2.39)
0.80	1.93 (2.15)	1.92 (2.62)	2.34 (2.31)	2.34 (2.15)	2.24 (2.25)	2.09 (2.17)	1.98 (2.09)	2.04 (2.32)	2.31 (2.33)
0.75	1.93 (2.18)	1.92 (2.56)	2.38 (2.29)	2.36 (2.13)	2.24 (2.22)	2.09 (2.17)	1.98 (2.12)	2.05 (2.31)	2.31 (2.29)
0.70	1.94 (2.29)	1.92 (2.53)	2.43 (2.26)	2.37 (2.14)	2.25 (2.21)	2.09 (2.19)	2.15 (2.16)	2.06 (2.26)	2.32 (2.30)
0.65	1.94 (2.36)	1.92 (2.50)	2.49 (2.23)	2.39 (2.14)	2.25 (2.19)	2.10 (2.19)	2.17 (2.18)	2.07 (2.22)	2.32 (2.27)
0.60	1.94 (2.44)	1.92 (2.43)	2.55 (2.16)	2.40 (2.14)	2.25 (2.18)	2.10 (2.21)	2.20 (2.21)	2.08 (2.16)	2.33 (2.27)
0.55	1.94 (2.54)	1.92 (2.39)	2.60 (2.13)	2.42 (2.12)	2.26 (2.15)	2.10 (2.21)	2.22 (2.25)	2.09 (2.12)	2.33 (2.25)
0.50	1.93 (2.63)	1.92 (2.37)	2.65 (2.10)	2.44 (2.12)	2.25 (2.14)	2.10 (2.25)	2.24 (2.28)	2.09 (2.08)	2.34 (2.21)

Note:1: Values in the bracket are for ACI:318-63 Method 3 for live load/dead load ratio = 1.0

2. A crosshatched edge indicates that the slab continues across or is fixed at the support, an unmarked edge indicate support at which torsional resistance is negligible.

Tables 5 and 6, and Method 3 of ACI: 318-63[6], will vary between 15 percent to 30 percent for the positive and negative moments. Coefficients given in Tables 5 and 6 will effect an overall economy of minimum of 10 percent.

3.4.4 Serviceability Checks

When the R.C. slab is designed by an ultimate strength theory like yield-line theory or modified Hillerborg's strip method, a separate check is necessary to ascertain the satisfactory behaviour of the slab at working load for cracking and deflection will make the slab unserviceable for an occupancy.

3.4.4.1 Cracking

The design process suggested in 3.4.3, uses 'Inelastic Analysis - Elastic Design' approach. With the 'Elastic Design (WSD),' the tension in the concrete is neglected. This is quite a conservative assumption at working loads, because for the same deformation the moment carrying capacity of the section is comparatively more than for the WSD; or for the same moment, deformations are less. (Fig. 3.9). This can be shown by $M_{cr} - M_{working} - M_{yield}$ relations as derived below with suitable assumptions.

Assuming

$$M_{yield} = A_{st}\, \sigma_{sy}\, 0.9\, d \qquad \ldots (3.44)$$

where A_{st} = Area of steel

d = effective depth of slab.

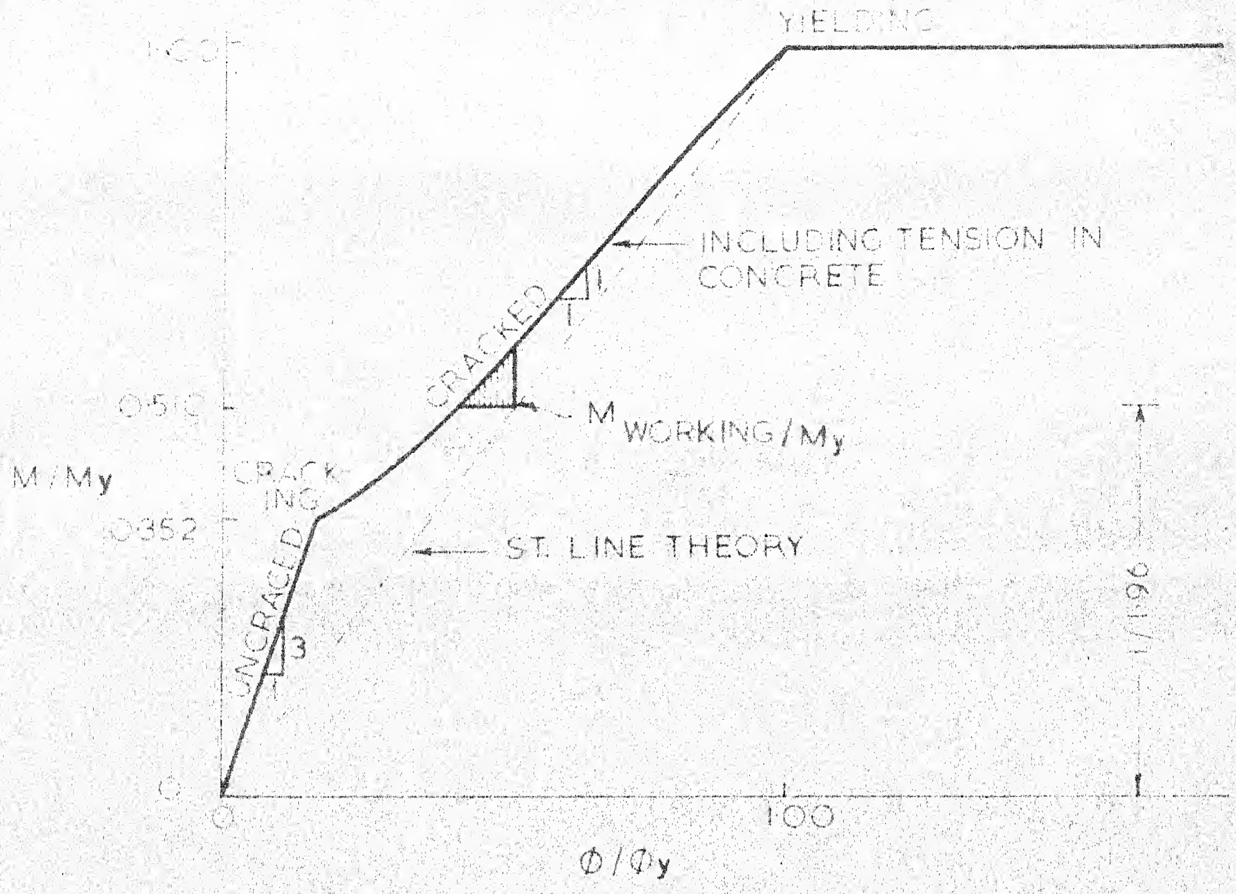


FIG. 3·9 MOMENT–CURVATURE RELATION FOR R.C. SLABS WITH LOW % OF REINFORCEMENT

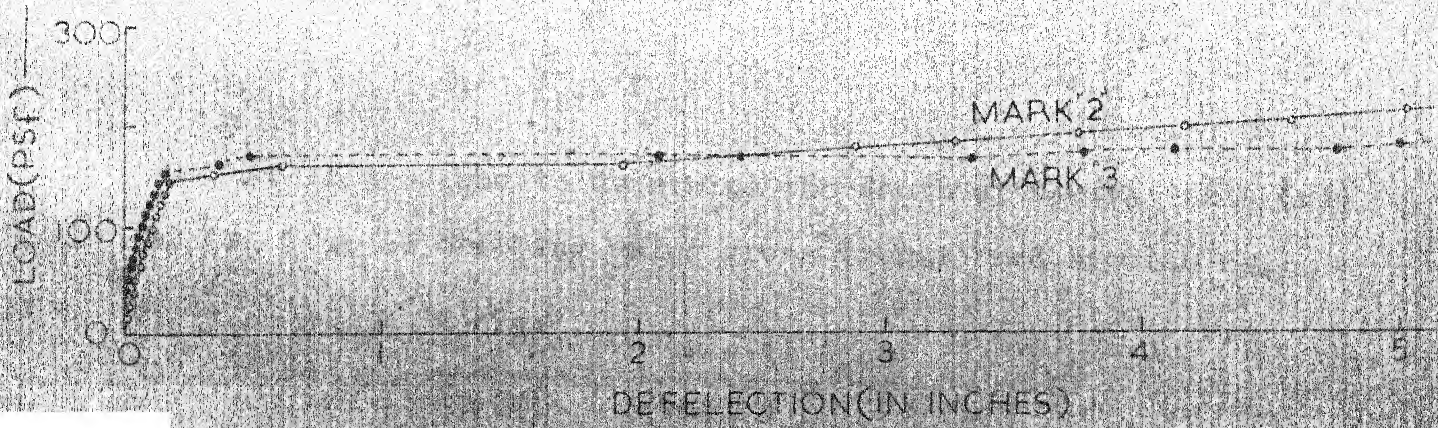


FIG. 3·10 LOAD DEFLECTION CURVES FOR ISOTROPICALLY (MARK 2) AND OPTIMALLY (MARK 3) REINFORCED SIMPLY SUPPORTED SQUARE SLABS (10'×10') (ROZVANY[56])

Now, from Equation (3.1), dropping K,

$$M_{working} = \frac{M_{yield}}{\sigma_{sy}/\sigma_{st}} \qquad \ldots \quad (3.45)$$

Using WSD,

$$d = \sqrt{\frac{M_{working}}{RB}} = \sqrt{\frac{M_{yield}\,\sigma_{st}}{\sigma_{sy}\;RB}} \qquad \ldots \quad (3.46)$$

where B = width of the slab

$$R = \frac{\sigma_{cb}\,n_1}{2}\left(1 - \frac{n_1}{3}\right)$$

σ_{cb} = Permissible bending stress in concrete in compression

n_1 = Ratio of a neutral axis for the balanced section (WSD) and the effective depth of slab.

Putting Equations (3.45) and (3.46); in (3.44),

$$A_{st} = \frac{M_{yield}\;RB}{0.81\,\sigma_{st}\;\sigma_{sy}} \qquad \ldots \quad (3.47)$$

Now, it is possible to determine the depth of neutral axis (nd) at the time of cracking using gross-transformed section as,

$$nd = \frac{\frac{B\,(\bar{k}d)^2}{2} + (m-1)\,A_{st}\,d}{B(\bar{k}d) + (m-1)\,A_{st}}$$

where $\bar{k}$ = Ratio of overall thickness of the slab to the effective depth (approximately = 1.2)

m = Modular ratio (= 2800 / 3 σ_{cb} for IS: 456-1964[7])

Putting Equation (3.47) in above,

$$nd = \frac{\frac{B\bar{k}^2}{2}\sqrt{\frac{\sigma_{st}}{\sigma_{sy}}\frac{}{RB}} + (m-1)\sqrt{\frac{RB}{0.81\sigma_{st}\sigma_{sy}}}}{B\bar{k}\sqrt{\frac{\sigma_{st}}{\sigma_{sy}}\frac{}{RB}} + (m-1)\sqrt{\frac{RB}{0.81\sigma_{st}\sigma_{sy}}}}\, d$$

$$n = \frac{\frac{\bar{k}^2}{2}\sqrt{\frac{\sigma_{st}}{R}} + (m-1)\sqrt{\frac{R}{0.81\sigma_{st}}}}{\bar{k}\sqrt{\frac{\sigma_{st}}{R}} + (m-1)\sqrt{\frac{R}{0.81\sigma_{st}}}} \qquad \ldots \quad (3.48)$$

Now, M_{cr} = Moment at cracking

$$= \sigma_{cr}\left[\frac{1}{2}Bd^2(\bar{k}-n)(\bar{k}-\frac{n}{3}) + (m-1)A_{st}(1-\frac{n}{3})d\right]$$

Using Equations (3.45) and (3.47) in above,

$$M_{cr} = \frac{\sigma_{cr}}{\sigma_{sy}}\left[\frac{1}{2}(\bar{k}-n)(\bar{k}-\frac{n}{3})\frac{\sigma_{st}}{R} + \frac{(m-1)}{0.9}(1-\frac{n}{3})\right]M_{yield} \qquad \ldots \quad (3.39)$$

where σ_{cr} = Tensile strength of concrete

For IS: 456-1964[7], Equation (3.45) takes the form

$$M_{working} = M_{yield} / 1.96 = 0.510\, M_{yield} \qquad \ldots \quad (3.50)$$

Illustrative Example

For $\sigma_{sy} = 2600$ kKg/cm^2

$\sigma_{st} = 1400$ Kg/cm^2

$\sigma_{cb} = 50$ Kg/cm^2 for M.150 (IS:456-1964[7])

establish the $M_{cr} - M_{yield}$ relation.

$m = 19$, assume $\bar{k} = 1.2$,

$R = 8.67$ for M.150[7] concrete and above mentioned steel stresses

then,

$$n = \frac{\frac{1.44}{2}\sqrt{\frac{1400}{8.67}} + 18\sqrt{\frac{8.67}{0.81 \times 1400}}}{1.2\sqrt{\frac{1400}{8.67}} + 18\sqrt{\frac{8.67}{0.81 \times 1400}}} = 0.64$$

Substituting this in Equation (3.49)

$$M_{cr} = \frac{12}{2600}\left[\frac{1}{2}(1.2 - 0.64)(1.2 - 0.21)\frac{1400}{8.67} + \frac{18}{0.9} \times (1-0.21)\right] M_{yield}$$

$$= 0.352\ M_{yield}$$

The results have been plotted in Fig. 3.9, for non-dimensional moment-curvature relationship, using the fact that usually the flexural rigidity in the uncracked region is approximately three times the value for the cracked region.

In Fig.3.9,hatched area shows the margin for serviceability or for increased moments by about 12 percent. It is also known fact that the thickness of slab will be less for ultimate strength design (USD) compared to the WSD. Thus, the WSD i.e. 'Elastic Design' is better for the serviceability.

If at a particular point in the slab, the elastic moment in the slab is more than that from the MHSM, then it will be taken case of, by the margin of the WSD (Fig. 3.9).

Moreover, the worst combinations[71] considered usually in the 'Elastic Analysis-Elastic Design' to compute the maximum bending moment at the critical sections due to live loads, will occur rarely during the life of the structure. Again, as discussed in chapter 1, the magnitudes of the live loads which is a random variable specified by the codes are quite conservative in the deterministic format. Thus the probability of the occurance of specified magnitude[10] and the actual worst combination[71] for the occupancy of the floor panel will be very less. So, for this probability, if the deformations are allowed, within the permissible crack width criteria (Fig. 3.11), for very few times during the life of the structure, it will be quite a rational and justified allowance.

3.4.4.2 Deflections

From the previous section it is quite clear that the thickness required by the WSD is more than that for the USD.

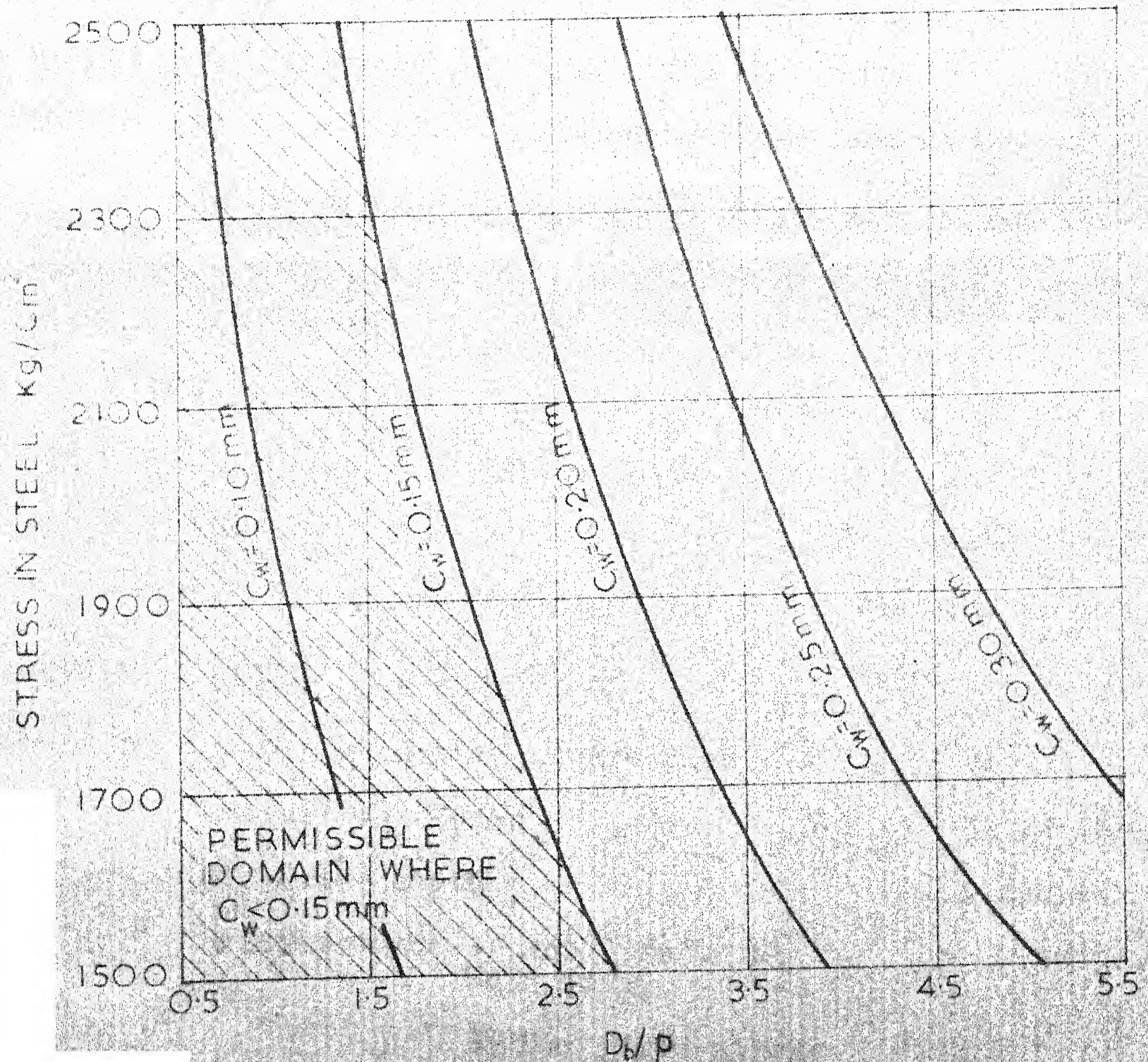


FIG. 3.11 CRACK WIDTH FOR PLAIN ROUND BARS WITH σ_{SY} = 2600 Kg/Cm² IN RELATION TO THEIR DIAMETER (IN Cm), % AND STRESS (RETCU[20])

With this and code provisions for the minimum thickness requirements from stiffness viewpoint are quite sufficient measures to guard against an excessive deflection at the working loads.

Except Baikov[3] in his book (following strictly U.S.S.R. code[2]) no other codes are specifying the limits on span/thickness ratios for the structural lightweight concrete slabs.

Tests conducted by Rozvany[56], Clyde and Sharpe[60] Armer[75] and Univ. of Ill. group[71,72] have been used to varify the limited deflection at working loads for the use of the MHSM for the uniform moment distribution across the span, and it is observed that the deflections at working loads are quite below the permissible values. Here, the case of the simply supported square slabs tested by Rozvany[56] is presented to illustrate and emphasize the point.

Rozvany's Tests

Details:	Simply supported square slab		10'x10'
	Overall thickness		3 1/4 inches
	Effective depth		2 inches
	Reinforcement	12 Nos. 1/4" at both ways.	10" c/c
		33 Ksi yield stress	
	Concrete	4000 psi cylinder strength at the time of test	
	Mark	2	
	q_u	149 lbs/sft.	

Computations for MHSM

For the factor of safety of 1.96 as discussed earlier,

$$q_w = 76.5 \text{ lbs/sft.}$$

Fig. 3.9 shows the load deflection curve for the slab tested with the abovementioned details. From Fig. 3.10, it is quite clear that at q_w = 76.5 lbs/sft, the deflection observed by Rozvany is very very less.

Thus, it is shown in 3.4.4.1 and 3.4.4.2 that the MHSM satisfies the serviceability check. However, it is very difficult to comment upon the cracking and deflections for MHSM, when the live load / dead load ratio is very high (more than 2), without any experimental evidences. It should be emphasized here, correctly that the previous discussion is strictly for short-time loading, and hence it is very difficult to comment upon the validity of the MHSM for the long-time loading and its effects like creep deformations etc.

3.4.5 Examination of MHSM in the light of theory of limit analysis

Due to the presence of strain-hardening of the reinforcement, the membrane action, the discrete location and the orthotropy of the reinforcement, none of the yield criteria known to-day cannot be satisfactorily and successfully applied to the cure of the R.C. slabs. (Fig. 3.12).

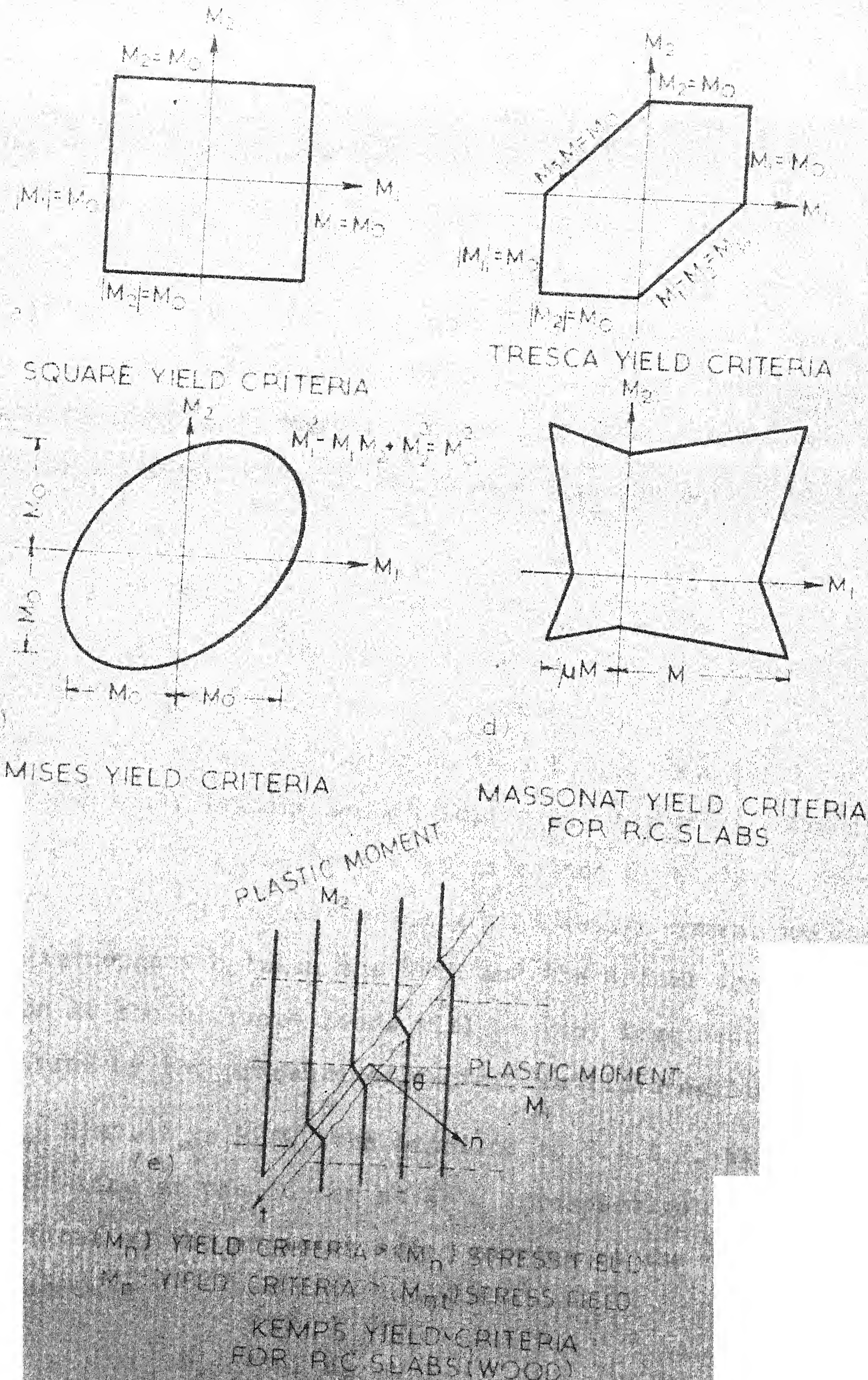


FIG. 3.12 VARIOUS YIELD CRITERIA

Johansen[23] in his yield-line theory assumes that the kinking of the reinforcement on an inclined yield-line, due to the presence of the twisting moments, is neglected as the yield-line theory employs an approximation based on the use of vectors. Even with such a basic assumption and approximation, the slabs designed by the yield-line theory have shown satisfactory performance, both at the working loads and the ultimate loads. This is due to the presence of factors outlined earlier in this section.

From Fig. 3.6, it seems prima facie that the yield criteria is violated with the MHSM moment distribution. It has been shown by Crawford that the moment distribution by Hillerborg's strip method is offering very poor lower bound on the collapse load, leading to the following statement. Moment distribution by Hillerborg's strip method is quite an oversafe distribution, giving excessively high design moment values.Thus the discrepancy between the MHSM and the actual moment distribution at the ultimate loads will be very less and it will be nullified by the presence of factors outlined earlier here.

Similar to MHSM, the practice in U.S.S.R. is to consider a yield-line at the corner at 45°, irrespective of the boundary conditions of the adjacent edges meeting at the corner[2,3]. It also assumes

$$M_s = 2 M_f$$

as assumed in the MHSM. For MHSM, advantages of the membrane action and the strain-hardening of reinforcement are neglected. In U.S.S.R.,[2,3] membrane (compressive, mainly) action is exploited and the collapse load is increased by 10 percent or 20 percent, or for the same collapse load, moments are reduced by 10 percent or 20 percent depending upon the boundary conditions of the panel. Fig. 3.13 shows the range of the coefficient of the orthotropy for the U.S.S.R practice,[2,3] Method 2 of IS:456-1964[7] and MHSM. Due to the difficulties encountered, in incorporating the membrane action and strain-hardening of the reinforcement, design procedures with the use of MHSM, neglects these factors partially on safe side.

As it has been discussed earlier in chapter 1, the 'Elastic Analysis - Elastic Design' as developed strictly by mathematical theory of elasticity exhibited excessive safety, when so-designed slabs were tested to failure. Compared to the theory of elasticity, if the theory of plasticity is used for an elasto-plastic material like R.C.slabs, satisfying plastic flow rule, equilibrium, mechanism conditions, yield criteria etc., then it will also lead to the same fate as the theory of elasticity had. Research workers are so strict in mathematical compliance of the various factors of the theory of plasticity. To sight an example, Wood[44] criticizes very strongly on Hillerborg's strip method (which is known to offer very poor lower bound) for the

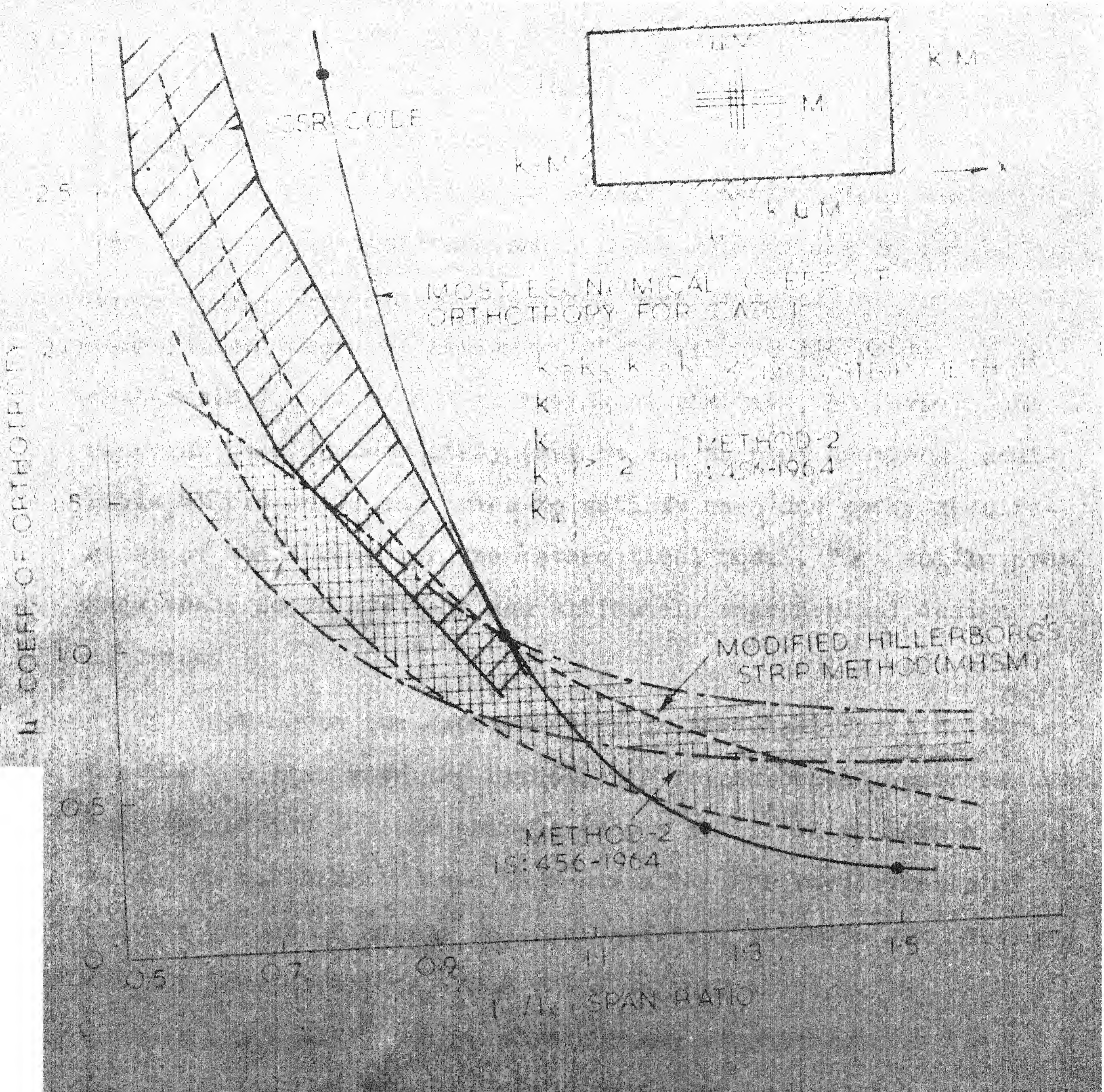


FIG. 3.13 COEFFICIENT OF ORTHOTROPIC REINFORCEMENT FOR DIFFERENT CODES AND MHSM

discontinuity in the moment field etc. That way, Johansen's yield-line theory[23] is also worth dumping in a garbage bin as the twisting or kinking of the reinforcement along the yield-line is neglected. Even with this discrepency the yield-line theory has proved excellent for the satisfactory behaviour at the working and ultimate loads and it is worth using. Nature has its own laws (yet unknown) for the case of the R.C, slab and does not know either theory of elasticity or theory of plasticity. Once the designer is strictly in the domain of the mathematical part of any of the above theories, he forgets the physical problem completely (may be due to poor knowledge available at present), and tries to satisfy each and every requirements of the theory for the mathematical model. This entire process leads to an oversafe and ultimately uneconomical design solution.

Looking to the uncertainties in the exact yield criteria for the R.C. slab with the orthotropic reinforcement, compressive membrane action and the plastic theory in its quite simple form in use in U.S.S.R.[2,3] When MHSM satisfies the requirements of minimum amount of safety (optimal safety), strength and serviceability considerations, MHSM which is very close to the method used in U.S.S.R.[2,3] can be straightaway recommanded for the design of the two-way R.C. slabs, with the restriction on span/ thickness ratio.

3.4.6 Curtailment of Negative Reinforcement

Codes do not, explicitly specify the curtailment of the negative reinforcement for different boundary conditions. Here, with the help of 'MHSM', recommendations are made for quantities L_1, L_2, L_3 and L_4 ; which represent the lengths of negative reinforcement for various cases as shown in Tables 4 and 6.

$$
\begin{aligned}
L_1 &= 0.21\, l_x + 0.5\, AL \\
L_2 &= 0.25\, l_x + 0.5\, AL \\
L_3 &= 0.21\, l_y + 0.5\, AL \\
L_4 &= 0.25\, l_y + 0.5\, AL
\end{aligned}
\qquad \ldots \quad (3.51)
$$

where AL is the anchorage length specified by the relevant code.

(Subject to a minimum of three times the effective depth of the slab, as recommended by the Swedish code[11].)

3.5 OPTIMUM SAFETY BY THE USE OF MHSM

The factor of safety, by the use of MHSM, is guaranteed to atleast the ratio σ_{sy}/σ_{st}, which satisfies the requirements of various codes of practice like IS:456-1964[7], ACI:318-63[6] etc., as can be seen from the comparative values of present design methods of these codes, from Table 2 and 7 and it is in parity with those of one-way slabs, two-way slabs, simply supported slabs etc. Acceptable behaviour at the working loads regarding cracking, deflection etc. is also assured by considering permissible crack width criteria, limit on span/thickness ratio and results of tests conducted by research workers. With the MHSM, yield

CHAPTER 4

OPTIMAL DESIGN OF TWO-WAY SLABS FOR STRENGTH

4.1 INTRODUCTION

In chapter 3, optimal design of two-way slabs for uniform safety was discussed with the use of straight reinforcements in strips or bands of specific width. The concept of moment redistribution due to plasticity was assumed. In this chapter, the design of two-way slabs such that all points in the slab yield simultaneously, is considered. This necessitates the use of variable reinforcement, although not necessarily orthogonal in curvilinear coordinates. This is done such that the minimum reinforcement is obtained both from elastic and plastic theories. Although such slabs may cost more due to fabrication if done individually, if multiple slabs of the same dimensions are to be manufactured (precast or cast in situ), considerable economy may be expected by use of mass production techniques. It should be noted however, optimal design for uniform strength is a particular case of optimal design with respect to safety. The converse is not necessarily true. It has been studied in the chapter 1, that the provisions of the codes for the two-way slabs result in the excessive safety. This is due to the recommanded uniform (uniform, in the sense of constant value) values of design moments

equal to the most critical values of moments (positive and negative) found by the elastic analysis. And the two-way slabs designed on this basis, due to the redistribution of forces exhibit the excessive safety and reserve strength. Rational safety (optimal safety) can be attained by avoiding this reserve strength and the cause, redistribution of forces. This is possible by having the strength varying from point to point and equal to the elastic (or an optimum plastic) stress field. Idea of having varying strength from point to point leads to the concept of uniform (or optimal) strength.

4.2 CONCEPT OF UNIFORM (OPTIMAL) STRENGTH

Uniform strength of the two-way slab is very similar to the uniform strength of a beam, and it is assured by the proportional variations in the resistance and the stress field (moments) at a point in the slab. For reinforced concrete slabs, with reasonable assumptions, a bending moment (M) in the particular direction at a point can be considered as a function of the thickness of the slab (t) at the point and the area of steel (A_{st}) at the point in the direction of the bending moment or it can be written as

$$M = M\ (t,\ A_{st}) \qquad \ldots \qquad (4.1)$$

For getting the slab of an uniform strength, t and A_{st} in Equation (4.1) should vary from point to point to match the stress field. But it is very difficult to vary both these factors

simultaneously due to difficulties encountered in the analysis and design. So, t or A_{st} is assumed uniform all over the plate from the stiffness (or economy) criterion or the ease with the use of fabric reinforcements respectively.

4.2.1 Thickness of Slab Varying: Reinforcement Uniform

When the reinforcement is uniform all over the slab, then in Equation (4.1) t will vary from point to point. This has been dealt by Brotchie[18]; by the elastic theory. Optimal plastic design, using Heymen's theorem and Prager and Shield's necessary and sufficient conditions, will be extremely difficult due to the complexies involved in the search of the virtual displacement field, which will satisfy uniform strain-field requirements for an optimal plastic design to exist.

The slabs with varying thickness from point to point will be architecturally excellent flooring systems, but it will increase the formwork cost and affect the economy very badly.

4.2.2 Thickness of Slab, Uniform: Reinforcement Varying

For the uniform strength of the slabs, if the thickness is kept uniform and determined from some other criteria like stiffness (to avoid excessive deflection at the working loads), economy etc.,; then the varying reinforcement (from point to point) layout with an orthogonal straight reinforcement can be had by the elastic theory and the optimal plastic design.

In certain cases curvilinear reinforcement layout have resulted in less volume than the one with orthogonal straight reinforcement layout. In some cases this type of layout may not be practical. It is worth noting here that the optimal plastic design offers the absolute minimum volume of reinforcement compared to the elastic theory, and the reinforcement layouts in both the cases will be totally different from each other.

Variable reinforcement layout will increase the labour cost, but this will be quite secondary and insignificant due to considerable material savings for the large repetitive work like mass housing projects etc. This layout, if found using the elastic theory will improve the performance the two-way slabs at the working loads and ultimate loads, as yielding occurs simultaneously at all points on the slab.

4.3 MINIMUM REINFORCEMENT USING ORTHOGONAL STRAIGHT REINFORCEMENT LAYOUT BY ELASTIC THEORY

It will be shown later in this chapter that the minimum volume of reinforcement is the one, resulted by providing the reinforcement varying from point to point. Here, in this study only orthogonal straight layout of the reinforcement will be considered, however a non-orthogonal reinforcement layout results in some case less volume of the reinforcements.

Assumptions and limitations of this minimum volume of the orthogonal straight reinforcement layout varying from point to point, are:

(1) Depth of the R.C. slab is kept constant and found by some other criteria like stiffness requirements, minimum cost etc.

(2) Constraints of the minimum required reinforcements for the temperature and shrinkage effects have been relaxed. However, Charret[65] and Kaliszky[63] consider this requirement in their optimization processes for two-way reinforced concrete slabs.

(3) Only one loading condition i.e. uniformly distributed load over entire area of the slab is considered to get the minimum volume of reinforcement and corresponding layout. If the load is other than the considered, then the minimum volume of reinforcement will not hold good. However, under various load combinations one can definitely find out the minimum volume, but the reinforcement may not be fully stressed. This very behaviour is similar to the three-bar truss problem solved by Schmidt[81].

With the above-made assumptions and restrictions, it is possible to find out the minimum volume of reinforcement. Wood[53] establishes that

> 'The reinforcement layout varying from point to point if it satisfies the elastic field of moments closely at all points on the slab, then this reinforcement layout yields near minimum (on conservative side) obtained by the limit analysis'

Here in this chapter, for finding out the minimum volume of reinforcements, two approaches will be considered. Firstly, using the elastic analysis for the slab (assuming $\nu = 0$), for all boundary conditions, the 'reinforcement layout' can be found using Wood's[52] approach. A case of simply supported square slabs is dealt with using this approach and its comparison is made with the available results by other theories. Secondly, using the concept of plastic minimum weight of Prager and Shield[59] and Heyman's[58] theorem (it is discussed in detail, later in this chapter), the search is made for getting an absolute minimum volume of reinforcements in two-way R.C. slabs. Here for R.C. slabs, it is necessary to satisfy the kinematic requirements as proposed by Rozvany[82]. Here in the second approach for the quick and easy (though correct) analysis, Hillerborg's[10] strip method is used. A case of clamped square slabs is tried out to show the relative merit of the second approach over the other solutions, already available.

So, in the next section, the general mode of attack for getting the field of moments according to the theory of elasticity is discussed for quick and immediate reference.

4.3.1 Moment Field According to The Theory of Elasticity

Layout in which reinforcement is varying from point to point in accordance with the elastic stress field can be had by

using the field of moments according to theory of elasticity and Wood's[52] requirements for most economical reinforcement layout.

For thin elastic plates with small deformations and assumptions of Kirchoff's (1850) hypothesis, the equilibrium equation can be written as:

$$D \nabla^4 w = q \qquad \ldots \qquad (4.2)$$

where $w = w(x,y)$

$= $ downward deflection

Taking

$$w(x,y) = \sum_{m=1,2}^{\infty} f_m(y)\, \phi_m(x) \qquad \ldots \qquad (4.3)$$

and this should satisfy boundary conditions for $x = 0$, a, and $y = \pm b/2$.

Exact solution of above Equation (4.2) is not possible for all types of rectangular panels shown in Fig. 4.1 except for type 1 and type 2.

Levy (1899) suggested taking the solution of the form

$$w(x,y) = \sum_{m=1,2..}^{\infty} Y_m(y) \operatorname{Sin} \frac{m\pi x}{a} \qquad \ldots \qquad (4.4)$$

From above it is very clear that it automatically satisfies the boundary conditions at $x = 0$, a. $Y_m(y)$ has to satisfy the boundary conditions at $y = \pm b/2$, as shown below.

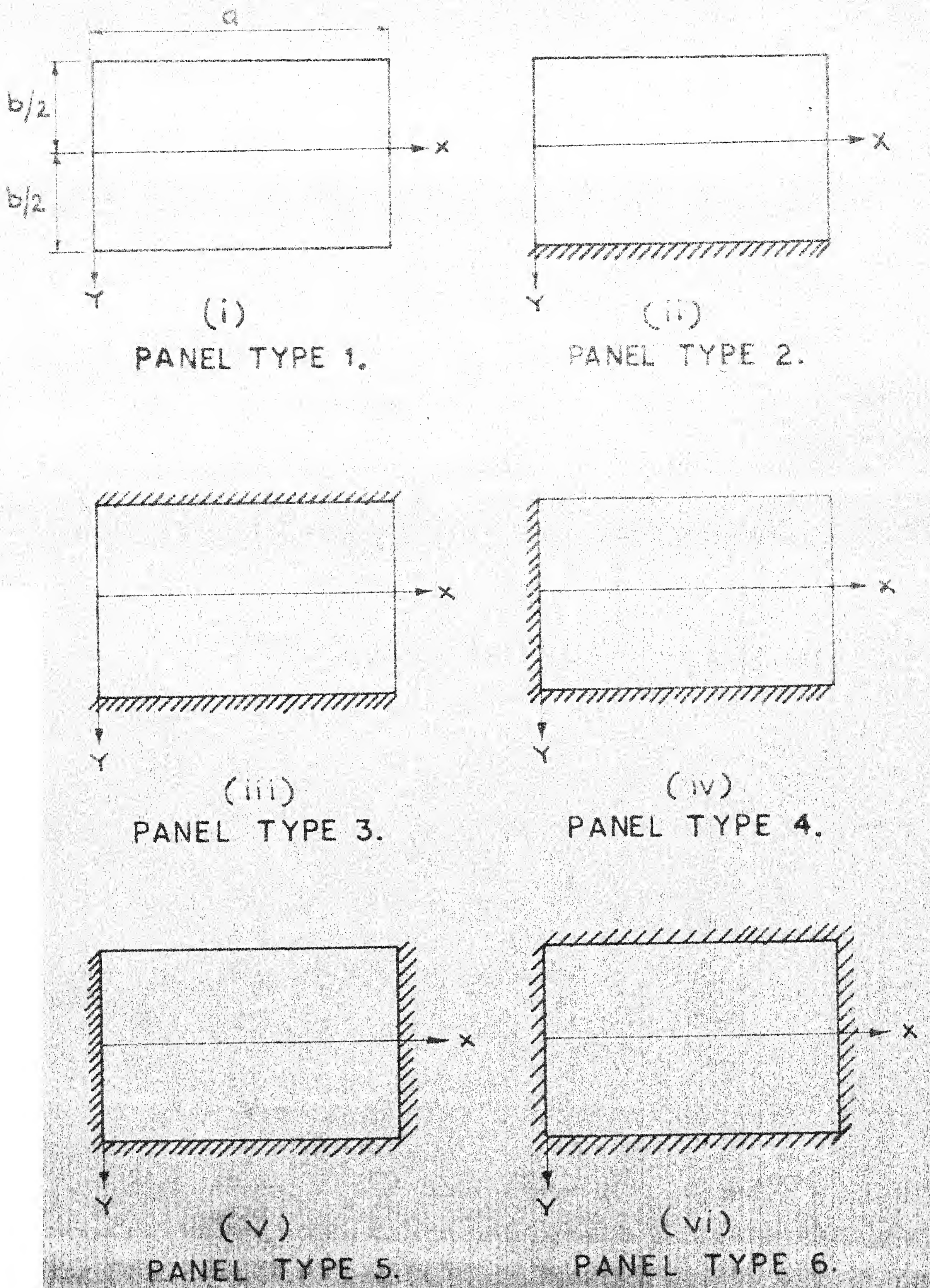


FIG.4.1 DIFFERENT TYPES OF PANELS.

—— SIMPLY SUPPORTED EDGE.

////// FIXED EDGE.

(i) Simply supported edges

$$\left.\begin{aligned} Y_m(\pm b/2) &= 0 \\ \frac{d^2 Y_m}{dy^2}(\pm b/2) &= 0 \end{aligned}\right\} \qquad \ldots \qquad (4.5)$$

(ii) Clamped edges

$$\left.\begin{aligned} Y_m(\pm b/2) &= 0 \\ \frac{d Y_m}{dy}(\pm b/2) &= 0 \end{aligned}\right\} \qquad \ldots \qquad (4.6)$$

It is to be noted here that $Y_m(y)$ is hyperbolic function in y direction of the form

$$Y_m(y) = Y_{mh}(y) + Y_{mp}(y) \qquad \ldots \qquad (4.7)$$

where

$$\begin{aligned} Y_{mh}(y) &= (A_m + B_m y) \operatorname{Cosh} \frac{m\pi y}{a} \qquad \ldots \\ &\quad + (C_m + D_m Y) \operatorname{Sinh} \frac{m\pi y}{a} \qquad \ldots \qquad (4.8) \end{aligned}$$

and

$Y_{mp}(y)$ for U.D.L. q

$$= \frac{2a^4 q}{D m^5 \pi^5} \left| \frac{m\pi y}{a} \operatorname{Sinh} \frac{m\pi y}{a} + 2\left(1 - \operatorname{Cosh} \frac{m\pi y}{a}\right) \right| \qquad (4.9)$$

Using Equations (4.8) and (4.9) in Equation (4.7), and putting it in Equation (4.4), one can get very easily displacement field.

Moment field then can be found from

$$\left.\begin{aligned} M_x &= -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \\ M_{xy} &= D(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \quad \ldots \quad (4.10)$$

where D = flexural stiffness of the plate

$$= \frac{Eh^3}{12(1-\nu^2)}$$

ν = Poisson's ratio

(ν = 0 for R.C. slabs)

h = thickness of the plate

E = Modulus of Elasticity of the plate material (i.e. concrete)

For panels type 3, type 4 zand type 5 exact solution is not possible to get. So, best approximate method suggested by Kontorovitch[83] will be dealt here. The method considers plate to be continuous in one direction and discrete in other direction. It is an approximate method, as it neglects corner forces.

For this method $\emptyset_m(x)$ is generally polynomial in x and it has to satisfy the geometrical boundary conditions only at x = 0, a while $f_m(y)$ has to satisfy following boundary conditions,

as the case may be:

(i) Simply supported edge

$$\left.\begin{aligned} f_k\,(\pm\, b/2) &= 0 \\ \frac{d^2\, f_k\,(\pm\, b/2)}{dy^2} &= 0 \\ k &= 1,2\ldots M \end{aligned}\right\} \qquad \ldots \qquad (4.11)$$

This gives 4M values of boundary conditions

(ii) Clamped edge

$$\left.\begin{aligned} f_k\,(\pm\, b/2) &= 0 \\ \frac{d\, f_k\,(\pm\, b/2)}{dy} &= 0 \end{aligned}\right\} \qquad \ldots \qquad (4.12)$$

Using Equations (4.11) and (4.12), one can get easily displacement field and subsequently moment field.

For panel type 6, Rayleight-Ritz suggested an approximate method and the solution can be found elsewhere[78].

Thus, moment field (M_x, M_y and M_{xy}) is known for all types of panels shown in Fig. 4.1.

4.3.2 Conditions for Most Economical Reinforcement Layout

Hillerborg[16] in his paper, indicates economical layout of orthogonal reinforcement. However, Wood[52] establishes rigorously, using Johonson's[23] stepped yield criteria, the most

economical layout of orthogonal reinforcements varying from point to point.

If at any point M_x, M_y, M_{xy} are known then principal moments M^*_x and M^*_y in x and y directions respectively can be found as shown below[52].

For bottom reinforcement

Generally

$$\left.\begin{aligned} M^*_x &= M_x + \left| M_{xy} \right| \text{ and} \\ M^*_y &= M_y + \left| M_{xy} \right| \end{aligned}\right\} \quad \ldots \quad (4.13)$$

If either M^*_x or M^*_y in above equation is found to be negative, then such a required value of reinforcement is changed to zero, as follows

either $$M^*_x = M_x + \left| \frac{M_{xy}^2}{M_y} \right| \text{ with } M^*_y = 0 \qquad (4.14)$$

or $$M^*_y = M_y + \left| \frac{M_{xy}^2}{M_x} \right| \text{ with } M^*_x = 0 \qquad (4.15)$$

It is to be noted here that if in these changed Equations (4.14) and (4.15), the wrong algebraic sign results for M^*_x and M^*_y, then no such reinforcement is required. Also, if both M^*_x and M^*_y are negative then no bottom reinforcement is required.

For Top Reinforcement

Generally $$\left.\begin{aligned} M^*_x &= M_x - \left| M_{xy} \right| \\ \text{and} \quad M^*_y &= M_y - \left| M_{xy} \right| \end{aligned}\right\} \quad \ldots \quad (4.16)$$

If either M^*_x or M^*_y in above equation is positive, then use the following Equations (4.17) and (4.18).

If $M^*_y \geq 0$ in Equation (4.16) then

$$M^*_x = M_x - \left| \frac{M_{xy}^2}{M_y} \right| \quad \text{with} \quad M^*_y = 0 \qquad \dots (4.17)$$

and if $M^*_x \geq 0$ in Equation (4.16), then

$$M^*_y = M_y - \left| \frac{M_{xy}^2}{M_x} \right| \quad \text{with} \quad M^*_x = 0 \qquad \dots (4.18)$$

No top reinforcements will be required, if in equations (4.17) and (4.18), the wrong algebraic sign results or both M^*_x and M^*_y in equation (4.16) are positive.

For every point in the plate it is necessary to check all the six Equations, (4.13) to (4.18) for safeguard against the possible omission of any reinforcement.

Finding out principal moments at every points in the plate is quite easy process. With the help of equations (4.13) to (4.18), but for computing volume of total reinforcement in the particular plate is really difficult process by these equations. For this Rozvany and Charret[82] suggested with the use of Nielsen's[27] linearised yield criteria, which is essentially same as that of Johnson's[23] stepped yield criteria, the expression for total reinforcement volume V_s for an R.C. plate, as shown below

$$V_s = \bar{\alpha} \iint_A \frac{1}{2} \left[|M_x| + |M_y| + 6|M_{xy}| + \Big| |M_x| - |M_{xy}| \Big| + \Big| |M_y| - |M_{xy}| \Big| \right] dxdy \quad \ldots \quad (4.19)$$

where $\bar{\alpha}$ = constant

4.3.3 'Isomoms' and their Importance

Isomom is defined here as a line joining equal moments M^*_x or M^*_y as obtained from Equations (4.13) to (4.18).

Isomoms can be best used to determine the theoretical curtailment of bars excluding the anchorge length required by the codes, reinforcement in form of strong and weak bands can be recommanded with the help of Isomoms. Using Equation (4.13) M^*_x and M^*_y have been computed for a simply supported square slab and variations have been shown in Fig. 4.2 with the help of 'Isomoms'.

4.3.4 Solution for Simply Supported Square Slabs

Now, expressing the total volume of reinforcements for the moment field shown in Fig. 4.2, as

$$V_s = \bar{c} \, \frac{qL^4}{24} \quad \ldots \quad (4.20)$$

then $\bar{c}$ = 1.42 in this particular case. In Table 8 values obtained by different research workers for orthogonal varying reinforcement layout, of $\bar{c}$ have been given for a comparative study.

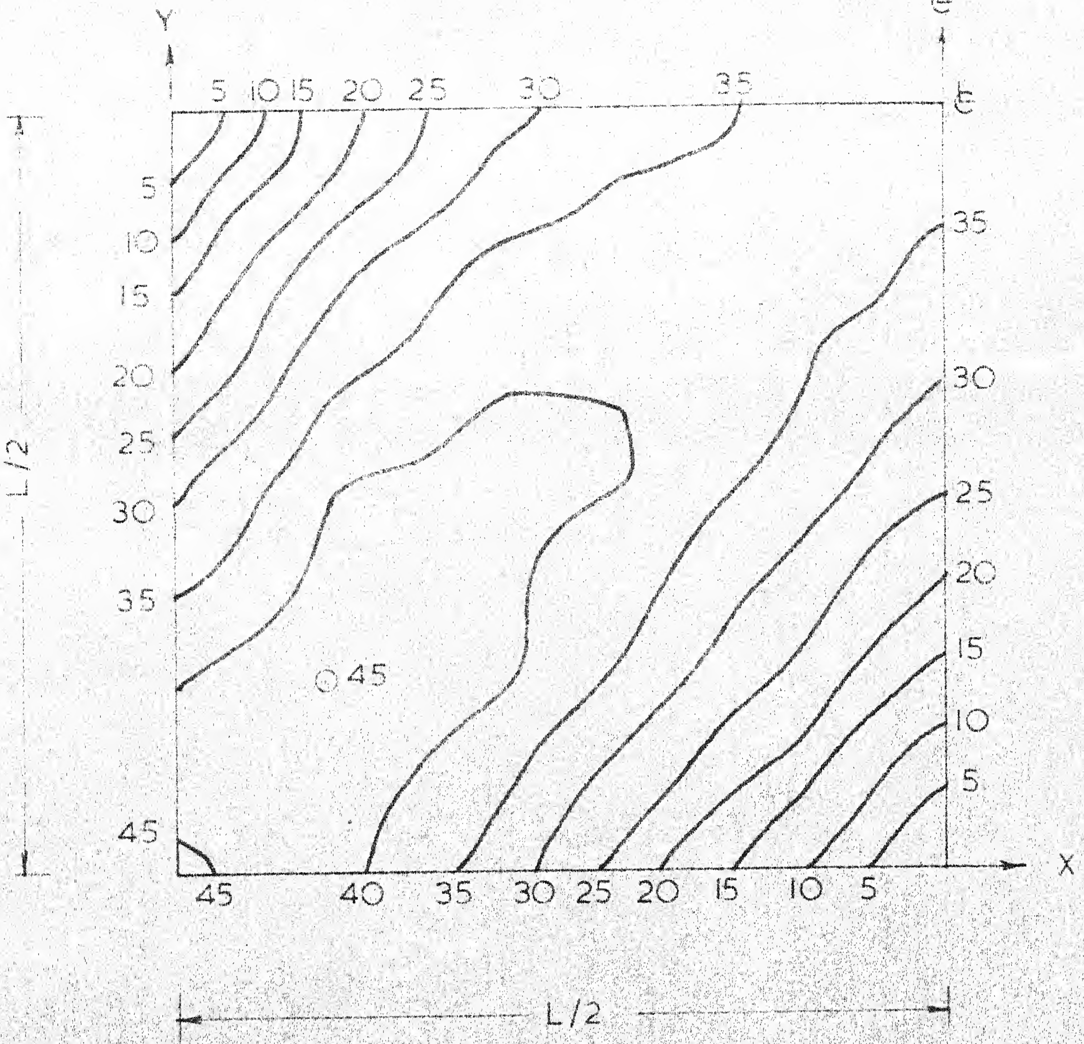


FIG. 4·2 ISOMOMS FOR M_x^* IN SIMPLY SUPPORTED SQUARE SLAB $(\times 10^{-3} \times qL^2)$

Table 8: DIFFERENT VALUES OF $\bar{c}$ FOR DIFFERENT APPROACHES

Values of $\bar{c}$, the coefficient to measure the total volume of reinforcements for simply supported square slab under uniformly distributed loads only.

(1)	One-way action or two-way action with constant equal distribution[55]	$\bar{c} = 2.00$
(2)	Membrane solution[55]	$\bar{c} = 1.75$
(3)	Curtailed optimum Hillerborg's strip method[60]	$\bar{c} = 1.5$
(4)	Optimum Elastic Solution[53]	$\bar{c} = 1.375$
(5)	Elastic Solution presented here	$\bar{c} = 1.42$
(6)	Absolute Minimum[55] (Optimum Plastic Design)	$\bar{c} = 1.25$
(7)	Uncurtailed Optimum Hillerborg's strip Method[60]	$\bar{c} = 2.10$
(8)	Uncurtailed yield-line theory solution[60]	$\bar{c} = 2.25$
(9)	IS: 456-1964 Method 2[7]	$\bar{c} = 2.39$
(10)	ACI: 318-1957 Method 3[6]	$\bar{c} = 1.66$
(11)	CP: 114-1957[8]	$\bar{c} = 3.90$

4.3.5 Comparative Study with Other Solutions

It can be seen from Table 8, that the minimum reinforcement (obtained by optimum plastic design) is the least and the value obtained for the optimum elastic solution obtained by Wood[53] near to the absolute optimum (approximately by 10 percent).

Detailed discussion focussing conservatism of the codes etc. can be found in 4.3.3.

4.4 MINIMUM REINFORCEMENT USING ORTHOGONAL STRAIGHT REINFORCEMENT LAYOUT BY OPTIMUM PLASTIC THEORY

4.4.1 Optimum Plastic Design

Here, firstly the concept of plastic minimum weight design, as proposed by Prager and Shield[59] considering uniform strains everywhere, in the structure to be optimized, will be shown by considering the case of an indeterminate beam. Heyman's[58] theorem with an assumption that bending moment is proportional to the cross-sectional area, can be used to arrive at the minimum weight solution (Fig. 4.3) with the assumption made above,

$$V = \int |M| \, dx \qquad \ldots \qquad (4.21)$$

where V = total wt. of the beam.

Of infinite number of designs corresponding to infinite number of bending moment diagrams, one will have the minimum weight.

Conditions, that minimum weight design has to satisfy, are:

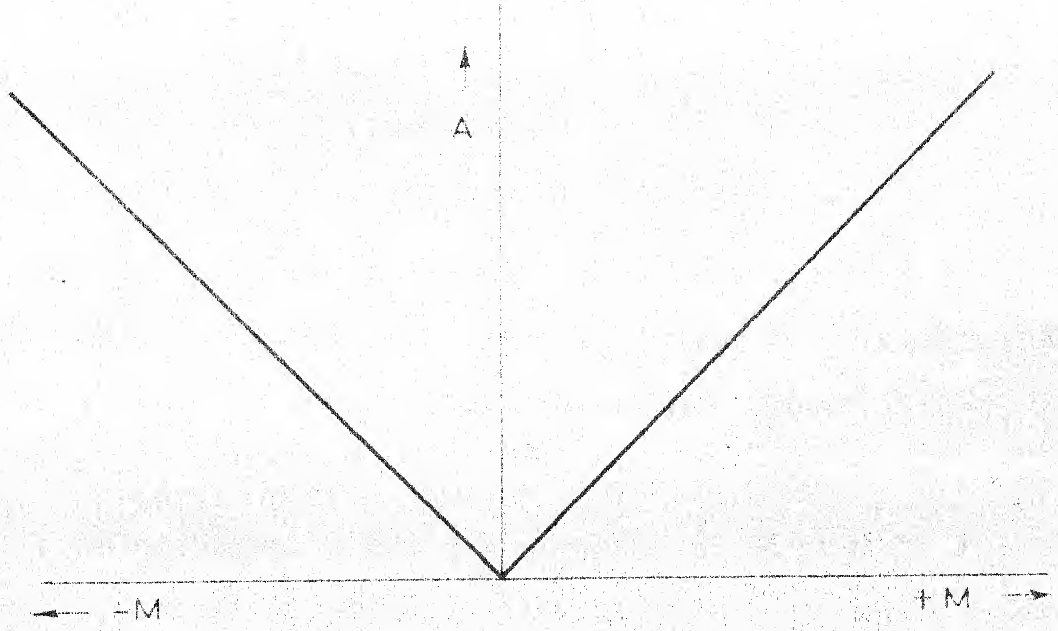


FIG. 4·3 RELATION BETWEEN M (BENDING MOMENT) AND A (CROSS SECTIONAL AREA). $A \propto |M|$

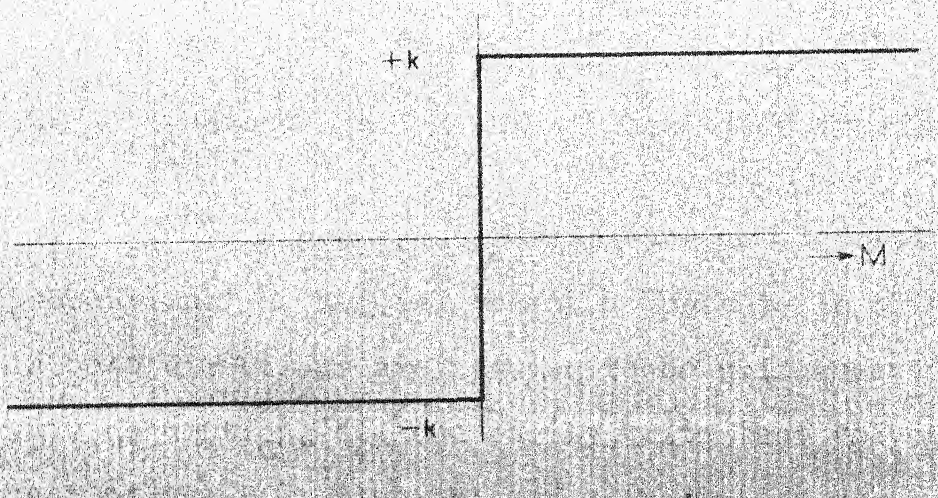


FIG. 4·4 MOMENT CURVATURE RELATION FOR KINEMATIC CONDITION FOR MIN. WT. DESIGN

(i) Equilibrium

(ii) Kinematic condition

(iii) Yield criteria should not be violated

(i) and (iii) can be easily satisfied, but for (ii) according to Heyman's[58] theorem the curvature k will be constant, and its relation with moment (for min. weight design) can be seen in Fig. 4.4.

Using Heyman's theorem, for the fixed beam of span L and with a uniformly distributed load of w per unit run, volume of the beam corresponding to absolute minimum weight with depth as constant, is computed in Fig. 4.5(a). For better comparison moment volumes for the prismatic beam, in the elastic range and, in the plastic range with w as collapse load is computed, as shown in Fig. 4.5(b) and (c) respectively.

This clearly established that absolute minimum volume is half of the plastic design, and is quite economical.

4.4.2 Necessary and Sufficient Conditions for Minimum (absolute) Reinforcement

In reinforced concrete slabs, area of reinforcement is proportional to bending moment. However, in this case depths in both directions are assumed to be uniform and constant. With this assumption, one can easily write that volume of reinforcement of the R.C. slab as:

$$V_s = \bar{\alpha} \iint_A \left[|M^*_x| + |M^*_y| \right] dxdy \qquad \ldots \qquad (4.22)$$

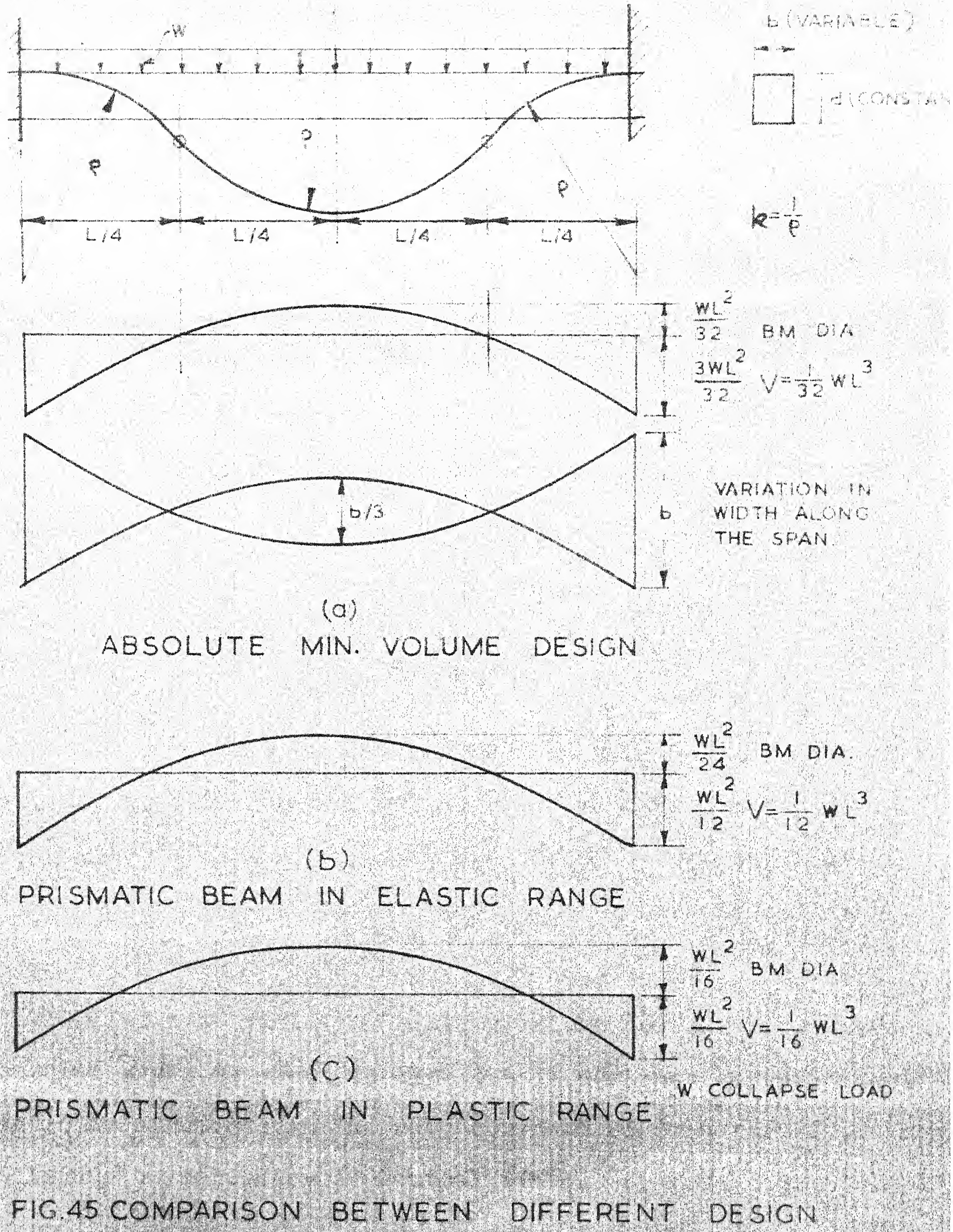


FIG. 45 COMPARISON BETWEEN DIFFERENT DESIGN SOLUTION OF FIXED BEAM LOADED WITH AN U.D.L.

or it can be expressed as shown by Rozvany[82] Equation (4.19).

Rozvany[82] establishes the necessary condition for the V_s to be minimum as follows:

$$\left.\begin{array}{lll} & \text{In region } R_1 : & M^*_x = 0 \text{ or } M^*_y = 0 \\ \text{and} & \text{in region } R_2 : & |M^*_x| = |M^*_y| \end{array}\right\} \quad \ldots \qquad (4.23)$$

These regions R_1 and R_2 should not be confused with R_x and R_y of Hillerborg's strip method. R_1 and R_2 regions can be seen in Fig. 4.6. Rozvany[82] with the help of Prager and Shield[59] also establishes the sufficient condition of kinematic requirements for the minimum reinforcement design solution, with the principal moments parallel to the reinforcement as:

$$\left.\begin{array}{ll} \text{either} & |k_x| = k \text{ and } |k_y| \leq k \\ \\ \text{or} & |k_y| = k \text{ and } |k_x| \leq k \end{array}\right\} \quad \ldots \qquad (4.24)$$

This concept of R_1 and R_2 by neutral lines (different from zero shear lines of Hillerborg's strip method) and the Kinematic conditions shown in Equation (4.24) will be used to optimize the reinforcement by the author for clamped square slab, later in this chapter.

Displacement field for minimum volume for simply supported rectangular slabs as shown in Fig. 4.6 it does not satisfy exact kinematic requirements in the neighbourhood of corners. This is possibly due to selection of neutral lines.

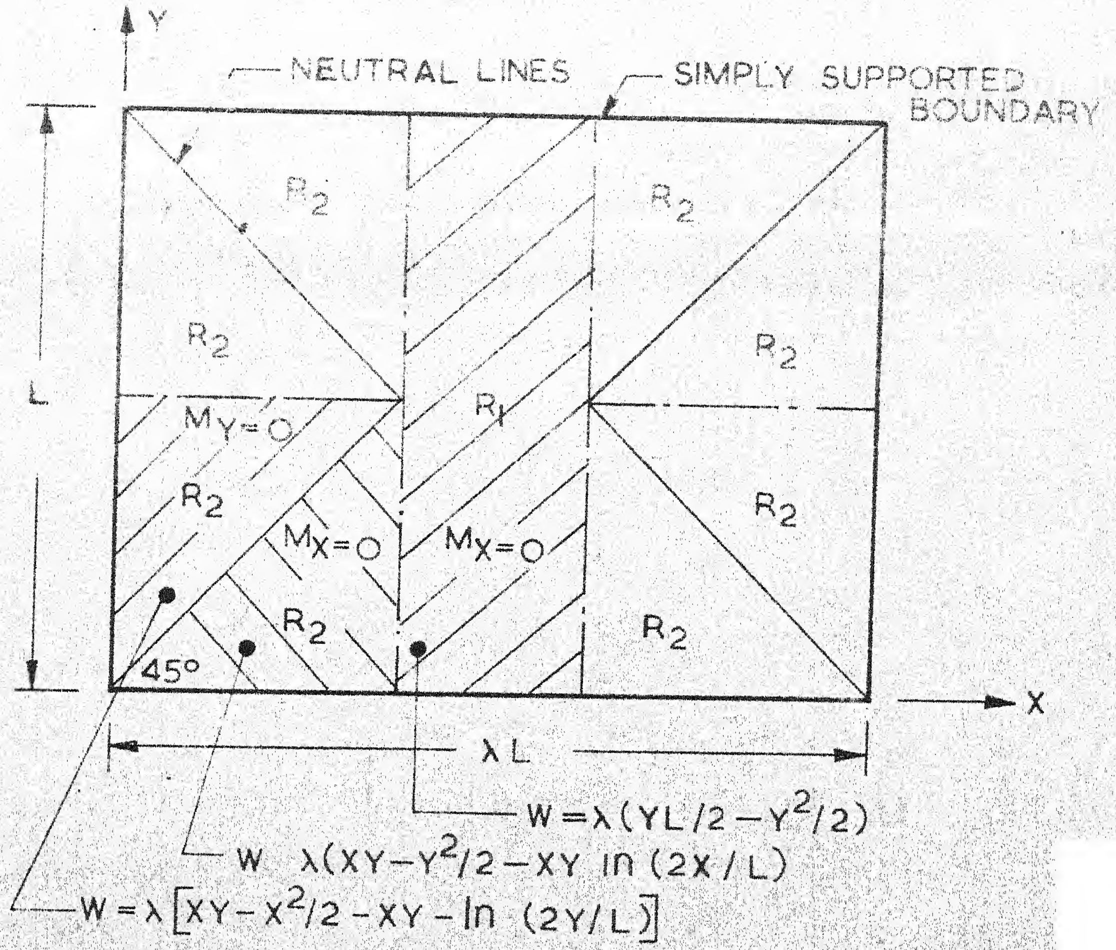


FIG. 46 REGIONS FOR MINIMUM VOLUME OF REINFORCEMENT FOR SIMPLY SUPPORTED RECTANGULAR SLAB (ROZVANY[82])

4.4.3 Yield Criteria

These have been already discussed in chapter 3.

4.4.4 Solution for Clamped Square Slabs

Clyde and Sharpe[60] have used Hillerborg's[16] strip method for all types of boundary conditions except all edges clamped for rectangular slabs.

Here in this paper, optimal plastic design of clamped square slab using Hillerborg's strip method and using necessary and sufficient conditions stipulated by Rozvany[82] is presented for uncurtailed and curtailed reinforcement layouts.

Also, in clamped square slab, neutral lines coincide with zero shear lines. Then,

$$\begin{aligned} R_x &\equiv R_2 \\ R_y &\equiv R_2 \end{aligned} \qquad \ldots \qquad (4.25)$$

Rozvany's[82] kinematic requirements (as shown earlier in the paper) are easily satisfied, if Hillerborg's strip method is used. Then

$$\begin{aligned} &\text{In } R_x: \quad |k_x| = k \text{ and } |k_y| \leq k \\ \text{and } &\text{in } R_y: \quad |k_y| = k \text{ and } |k_x| \leq k \end{aligned} \qquad \ldots \qquad (4.26)$$

unlike, simply supported rectangular or square slabs,

kinematic requirements are exactly satisfied at corners. This phenomena is due to absence of twist at corners of clamped square slab. Now, absolute minimum volumes of reinforcement (of course proportional to volume of moments) for curtailed reinforcements and uncurtailed reinforcements, both subject to load transfer constraint of the strip method, are considered here. If the reinforcement is such that it can vary from point to point, in both directions (i.e. both in length and spacing, in finite strip simultaneously) then such a layout offers a curtailed optimum solution. If, in finite strip width length of bars do not vary but spacings do vary, then the reinforcement layout results in an uncurtailed optimum solution.

Curtailed optimum:

Considering two different strips, AA (valid for range $0 \leq x \leq L/4$) and BB (valid for range $L/4 \leq \bar{x} \leq L/2$) of the clamped square slab as shown in Fig. 4.7, minimum moment volumes for unit width of strip AA and BB have been computed and shown in Fig. 4.8(i) and (ii) respectively.

Now, total minimum volume of moments can be computed as

$$V_m = 4 \int_0^{L/4} \frac{qx^3}{3}\, dx + 4 \int_{L/4}^{L/2} \frac{qL^3}{192} \Big[(8 \frac{\bar{x}}{L} - 1) + 4 (1 - \frac{\bar{x}}{L}) (4 \frac{\bar{x}}{L} - 1)^2 \Big] d\bar{x}$$

$$= \frac{3}{192} qL^4 \qquad \ldots \qquad (4.27)$$

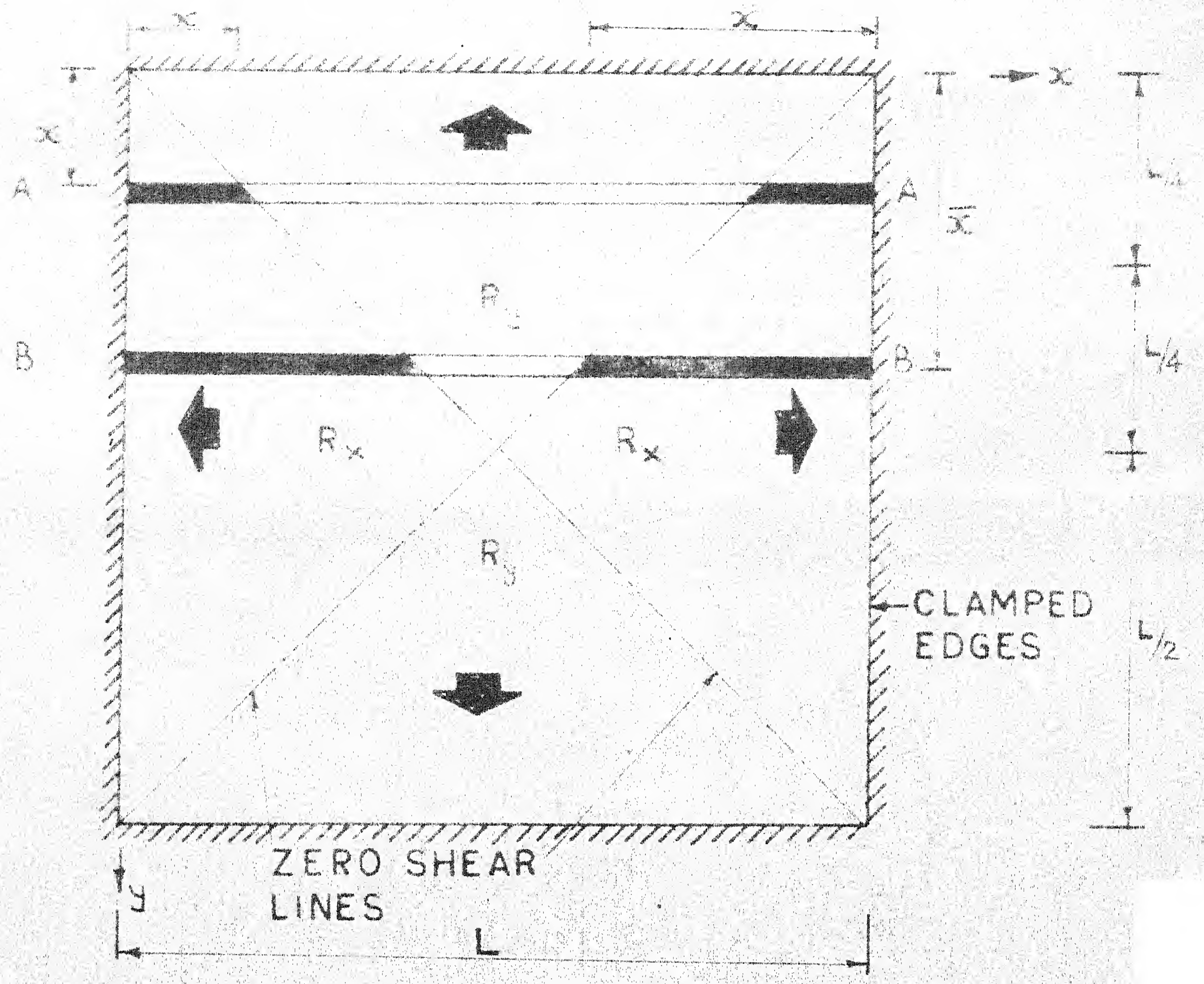
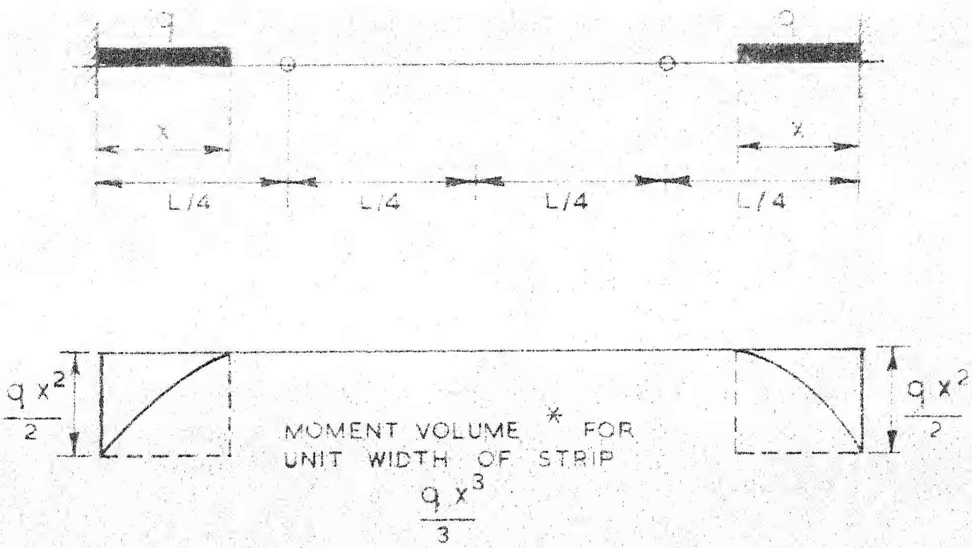
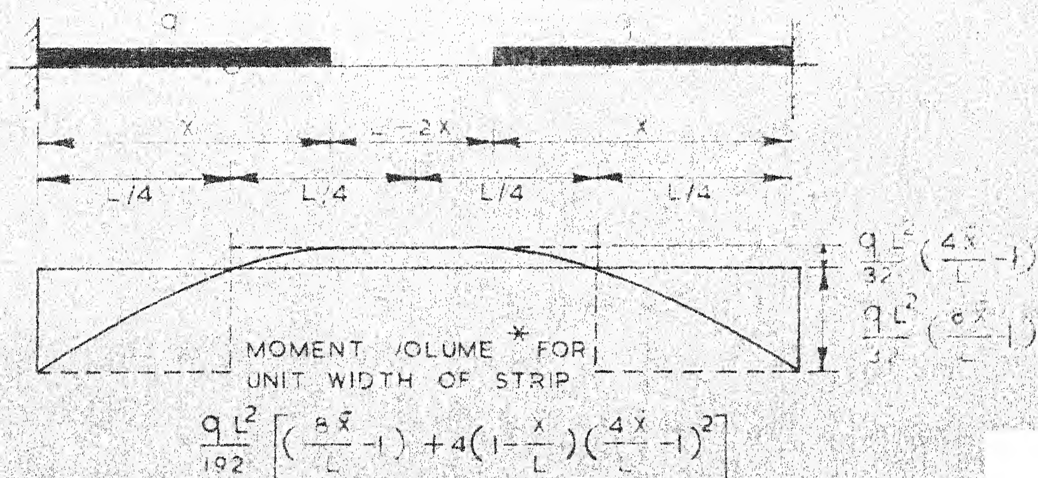


FIG.4.7 HILLERBORG'S STRIPS IN CLAMPED SQUARE SLAB



(i)

STRIP AA FOR $0 \leq x \leq L/4$



(ii)

STRIP BB FOR $L/4 \leq x \leq L/2$

FIG.48 MOMENT VOLUMES CORRESPONDIND TO MINIMUM MOMENT VOLUME CONDITIONS FOR STRIP AA & BB.

* FOR CURTAILED REINFORCEMENTS

Uncurtailed optimum:

Here, total minimum volume of moments can be computed using moment diagrams (dotted and uniform) of Fig. 4.8 (i) and (ii).

$$V_m = 4 \int_0^{L/4} qx^3 \, dx + 4 \int_{L/4}^{L/2} \frac{qL^3}{32x2} \left(4 \frac{\bar{x}}{L} - 1\right)^2 d\bar{x}$$

$$+ \quad 4 \int_{L/4}^{L/2} \frac{qL^3}{32x2} \left(8 \frac{\bar{x}}{L} - 1 \right) d\bar{x}$$

$$= \frac{31}{12 \times 64} \, qL^4 \qquad \ldots \quad (4.28)$$

Now, expressing

$$V_m = c \, \frac{qL^4}{64} \qquad \ldots \quad (4.29)$$

where c is a constant for different approaches. Different values of c can be found in Table 9 for better comparison.

4.4.5 Comparative Study with other solutions

Similar to the study of simply supported square slab, here also, the reinforcement volume obtained by using the optimal plastic design concepts is the least and absolute minimum. The corresponding value obtained by the elastic theory is quite high compared to the absolute minimum.

4.5 DISCUSSION

It can be seen from the study that the reinforcement layout corresponding to the total minimum volume of the reinforcements, is very complicated and highly unpractical. But

Table 9: DIFFERENT VALUES OF c FOR DIFFERENT APPROACHES

Values of c, the coefficient to measure the total volume of reinforcements for a clamped square slab under uniformly distributed loads only.

(1)	Curtailed Optimum Hillerborg's Strip Method	c = 1.00
(2)	Uncurtailed Optimum Hillerborg's Strip Method	c = 2.58
(3)	'Discrete Elastic Solution' based on Swedish Regulations[11] with M_{xy} = 0 everywhere in the slab (Wood[53])	c = 2.56
(4)	Wood's[53] approximate solution based on finite difference technique	c = 1.34
(5)	IS: 456-1964[7] Method 2	c = 4.96
(6)	ACI:318-1963[6] Method 3	c = 4.31
(7)	Modified Hillerborg's Strip Method[47]	c = 3.52
(8)	Yield line theory[53] solution	c = 4.00
(9)	CP 114: 1957[53]	c = 6.93

this type of layout will save considerably where labour is very cheap and/or precasting work and/or more repetitive type of work (e.g. multi-storey structure with the same module for panel of slab). This layout neglects the requirements of minimum reinforcements required for the temperature and shrinkage effects on the slab. Depth of the slab is to be fixed by arbitrary measure, but in next chapter it will be shown that having known the maximum principal moment, this can be found for the minimum cost criteria, however it has to satisfy the minimum thickness requirements for the slabs.

Comparatively with less amount of reinforcements placed in proper place and length, with little increase in labour cost, the behaviour of the reinforced concrete slab is very much improved with respect to cracking. This is an observed fact by Rozvany[56], who tested so optimized simply supported square slabs. He also observed that the deflection at working load was less than that for the slab designed by the yield-line theory for the same thickness of the slab.

One more thing, worth noting here is that, so optimized slabs fail like the plastic hammock. This behaviour is due to the fact that every point in the slab yields at the same time, meaning thereby that there is no redistribution of forces in the optimally reinforced slabs.

Now, studying the results tabulated in Table 8 and Table 9, it is clear that most of the codes provide for 3 to 7

times the minimum volume of the reinforcements. The conservatism of the codes seems to be due to oversimplification of the optimum requirements. Of course, it is very difficult to find out the minimum reinforcement layout, capable enough to take care of all possible load combinations, but it is not impossible. Reinforcement layout with due and proper recognition to curtailment and selection of proper bands (other than traditionally defined bands) and some averaging out on conservative side will remove this discrepancy of the codes.

It is worth mentioning here, that by selecting proper reinforcement layout economy can be considerably affected, but the simplicity of the layout remains. Rozvany[62] showed that in a design of circular footing, the volume of the square mesh isotropic reinforcement in the footing is three times that using a spiral reinforcement layout (which is not at all difficult and unpractical). Thus, if proper care is taken in selecting a right type of orthogonal reinforcement layout in the case of two-way R.C. rectangular slabs, by a proper band width, conservative averaging in it, a proper curtailment selection etc. economy can be considerably affected and yet the behaviour of the slab will improve in general.

4.6 BEHAVIOUR OF SLABS WITH MINIMUM REINFORCEMENT LAYOUT

Experimental evidences show that the two-way slab with the minimum reinforcement either by the elastic theory or by the optimum plastic design assures the satisfactory and acceptable

behaviour both regarding the serviceability at the working loads, and the warning message giving ductile behaviour at the ultimate loads.

In particular, Rozvany[56] tested simply supported square slabs under the uniformly distributed load one with the absolute minimum reinforcement (layout corresponding to the optimum plastic design) and the other with uniform reinforcement designed by the yield-line theory. Fig. 3.10 shows load-deflection curves for both the cases and it can be seen that the behaviour of the slab with the optimal strength is better.

Thus, optimal strength design of the two-way slabs under one particular loading condition only, assures required serviceability and optimal safety, however the economy is affected.

4.7 USEFULNESS OF MINIMUM REINFORCEMENT LAYOUTS AND ECONOMY AFFECTED

Use of the varying reinforcement layout in its complex form or the proper selection of strong or weak bands from the study of 'Isomoms', can be made in the projects involving a repetitive type of construction and considerable capital investments like mass housing projects or pre-fabrication etc. Due to the varying reinforcement layout for such type of projects provides a basis making trade-off between the increase in the labour cost and the savings in the reinforcing steel.

Rozvany[62] showed 67 percent savings in the reinforcement for the circular footing for a column with an axial load, by adopting curvilinear (spiral in this case) reinforcement layout rather than an isotropic and orthogonal straight reinforcement layout. For large number of repetitions, spiral reinforcement layout, though costly for a single footing, will be quite an economical proposal. Thus, for the repetitive type of work, the economy, one of the basic values, will not be affected as is feared, but on the contrary will reduce the cost to little extent.

CHAPTER 5

OPTIMAL DESIGN OF TWO-WAY R.C. SLABS FOR ECONOMY

5.1 INTRODUCTION

In any reinforced concrete structure where the two-way slabs have been used as a structural flooring, total cost of the two-way slabs is nearly 40 to 45 percent of the total structural cost. This stresses the necessity of the attention to the problem with respect to economy. Economy is used here in the sense of the first cost. If proper care is not taken to design the two-way slab for minimum cost, which satisfies all structural and design requirements, it will increase the cost by a minimum of 5 percent. Then, due to the proper optimal cost design not only save this minimum 5 percent directly, but also save on designs of a supporting structure i.e. beams, columns, foundations etc., indirectly. The object of this chapter is to forgo these indirect savings and to focuss attention on the economics of the problem of the two-way slabs. Examples are given of the possible economy in design of slabs using relevant codes of practice or use of Modified Hillerborg's Strip Method (MHSM) in this chapter and chapter 7, with respect to the prevailing cost in India, in general. A review of the mathematical formulation used for optimal cost studies by various authors is also given. A computer programme for optimum cost studies has been developed and used for drawing conclusions.

A flow chart of the programme is given.

The problem of finding the design parameters corresponding to the optimal cost design, can be solved by minimizing the cost function subject to problem solving constraints and code restrictions.

5.2 COST FUNCTION AND CONSTRAINTS

Cost of the two-way R.C. slabs per unit area can be considered as made up of cost of concrete per unit area, cost of shuttering per unit area and reinforcement cost per unit area. Some research workers assume the shuttering cost to be constant, but it will not be so in the practice. For example, unit cost of the shuttering for $3^m \times 3^m$ panel and $6^m \times 6^m$ will not be same. In later case it will be more than previous one. So, it is assumed that the cost of the shuttering is directly proportional to the thickness. It is worth noting, here, the tendency of the contractors of India, to quote for the concrete only, including shuttering for two-way slabs, which means a thicker slab will be costlier than a thinner slab. Now, the cost of the two-way R.C. slabs per unit area is assumed to be made up, of the cost of the concrete (including shuttering) per unit area and the cost of the reinforcement per unit area, or it can be expressed as:

$$C = C_c \, Q_c + C_s \, Q_s \qquad \ldots \quad (5.1)$$

where C is the overall cost of the two-way slab per unit area (Rs/m^2)

C_c is the cost of the concrete in shuttering per unit volume (Rs/m^3)

C_s is the cost of the reinforcing steel per unit wt. (Rs/Kg)

Q_c is the volume of the concrete of the two-way slab per unit area (m^3/m^2)

and Q_s is the reinforcing steel of the two-way slab per unit area (Kg/m^2)

In above Equation (5.1), it is quite clear that

$$Q_c = Q_c(t) \qquad \ldots \quad (5.2)$$

where t is the overall thickness of the two-way slab.

Also, it can be assumed correctly for the known size of the panel and the loads,

$$Q_s = Q_s\,(A_{stmax}) \qquad \ldots \quad (5.3)$$

but,

$$A_{stmax} = A_{stmax}\,(M_{max}) \qquad \ldots \quad (5.4)$$

where A_{stmax} is the area of the reinforcement per unit width for the moment M_{max} (Cm^2/m) and, M_{max} is the absolute maximum moment in the panel. $(Kg.cm/m)$

Then, using Equation (5.3) and (5.4),

$$A_{stmax} = A\,Q_s \qquad \ldots \quad (5.5)$$

where A is the constant depending upon the panel size and the load.

Also, it can be shown that

$$A_{stmax} = B \frac{M_{max}}{t} \qquad \ldots \quad (5.6)$$

where B is the constant depending upon the permissible stress in steel or yield stress, concrete strength, overall depth of slab etc.

From **Equation** (5.5) and (5.6),

$$t\,Q_s = M_{max} \frac{B}{A} = \bar{B} \qquad \ldots \quad (5.7)$$

So-called constant $\bar{B}$ will have different values for different thicknesses, and this behaviour is troublesome in the problem solving. If $\bar{B}$ is assumed to be constant, then the problem is:

$$\left.\begin{aligned} &\text{Minimize} \quad C = \bar{C}_c\, t + Q_s\, C_s \\ &\text{Subject to} \quad Q_s t \geq \bar{B} \\ &\qquad\qquad\quad t \geq t_{min} \\ &\qquad\qquad\quad Q_s \geq Q_{s\,min}. \end{aligned}\right\} \qquad \ldots \quad (5.8)$$

where $\bar{C}_c$ is the new cost constant for the concrete

t_{min} is the minimum thickness required by the relevant code and

$Q_{s\,min}$ is the minimum reinforcements required by the relevant code.

One of the constraints in Equation (5.8) is non-linear. If $\bar{B}$ is the constant in the real sense, then the problem can be easily solved by mathematical programming, but it is not constant. For assumption of it being constant for the classical solution of the problem, the entire economy considerations will be swept away. So, the best mode of attack is the one by an exhaustive search for minimum cost design parameters, changing $\bar{B}$ every now and then. This is done for the working stress design method by Thakkar and Sridhar Rao.[67]

5.3 VARIOUS DESIGN THEORIES FOR TWO-WAY R.C. SLABS

Now it is necessary to specify, which design theory will be used out of the two as:

(1) Elastic Theory (or known as Working Stress Method)

(a) Elastic Analysis-Elastic Design
Under this head, all codes specify moment coefficients for working loads and the design is to be carried out by the elastic theory.[6,7,8]

(b) Inelastic Analysis - Elastic Design
This is most popular in Scandanavian countries like Sweden, Denmark etc. Thakkar and Sridhar Rao[47] have suggested moment coefficients for Indian Standard Code of practice[7], which ensures strength, serviceability, optimum and desired safety and economy, too, by taking advantages of the inelastic

analysis and the elastic design. This approach is presented in chapter 3.

(2) Ultimate Strength Theory

(a) Elastic Analysis - Inelastic Design

This approach will give advantage of the inelastic design. Redistribution of moments, membrane action etc. are totally ignored. Here a seperate check for the serviceability (cracking, deflection) is necessary.

(b) Inelastic Analysis - Inelastic Design

Johnson's[23] yield line theory uses this concept.

Here in this chapter, procedures for getting optimal cost design parameters for 1(a) and 2(a) will be discussed. With little or no modifications, procedures for 1(b) and 2(b) can be prepared.

Design procedures for 1(a) and 2(a) discussed in the following sections, have to satisfy the relevant requirements of the code for the minimum thickness of the two-way slab, the minimum percentage of the reinforcements, the maximum spacing between two-bars and the minimum clear cover to the reinforcements.

5.4 OPTIMAL COST DESIGN BY ELASTIC THEORY

Various codes of practices for reinforced concrete[6,7,8] specify the moment coefficients for the design of the two-way slabs, working loads. Here in this section Method 2 of

IS: 456-1964[7], without the provision of torsion reinforcements, will be discussed.

Using the moment coefficients for a given span ratio and boundary conditions, one can easily find out M_x, M_y, M_x' and M_y' design moments, from

$$\left.\begin{aligned} M_x &= \frac{q l_x^2}{m_x} \\ M_y &= \frac{q l_y^2}{m_y} \\ M_x' &= \frac{\alpha_x \, q l_x^2}{J_1}, \text{ and} \\ M_y' &= \frac{(1-\alpha_x)\, q l_y^2}{J_2} \end{aligned}\right\} \quad \ldots \quad (5.9)$$

where J_1 and J_2 may be 8, 10 or 12 depending upon the boundary conditions of the panel, M_x and M_y are the design span moments in X and Y directions respectively, M_x' and M_y' are the design support moments in x and y directions respectively, q is the design load,

l_y and l_x are the effective spans of the panel in y and x directions respectively,

α_x, m_x, m_y are the coefficients specified in Method 2 of IS:456-1964[7] without the provision of torsion reinforcements.

Absolute maximum beinding moment can be found from Equation (5.4). as

$$M_{max} = M_{ax} \left(M_x, M_y, |M'_x|, |M'_y|\right) \quad \ldots \quad (5.10)$$

It is observed that the solution is optimal with respect to cost, when an assumed overall thickness of the slab matches with the computed overall thickness of the slab, however subject to the constraint of the minimum thickness requirements of the code (Fig. 5.1). This is best illustrated by considering an illustrative example.

5.4.1 Illustrative Example

Find out the minimum cost design solution using the coefficients of Method 2 (without the provision of torsion reinforcements) of IS:456-1964[7], for two-way simply supported square slab with the side of 5 m., for the live load of 1000 Kg/m^2 . Use M.150 concrete and Grade I mild steel with σ_{sy} = 2600 Kg/cm^2. The minimum reinforcement, the minimum cover, the maximum spacing of the reinforcements and the minimum thickness shall be governed by relevant clauses of IS:456-1964. Take C_c = 180.00 Rs/m^3 and C_s = 1.20 Rs/Kg.

Cost function of Equation (5.8) for this example is

$$C = 1.80\ t + 0.048\ S_w \quad \ldots \quad (5.11)$$

where $S_w = Q_s$ x Area of the panel

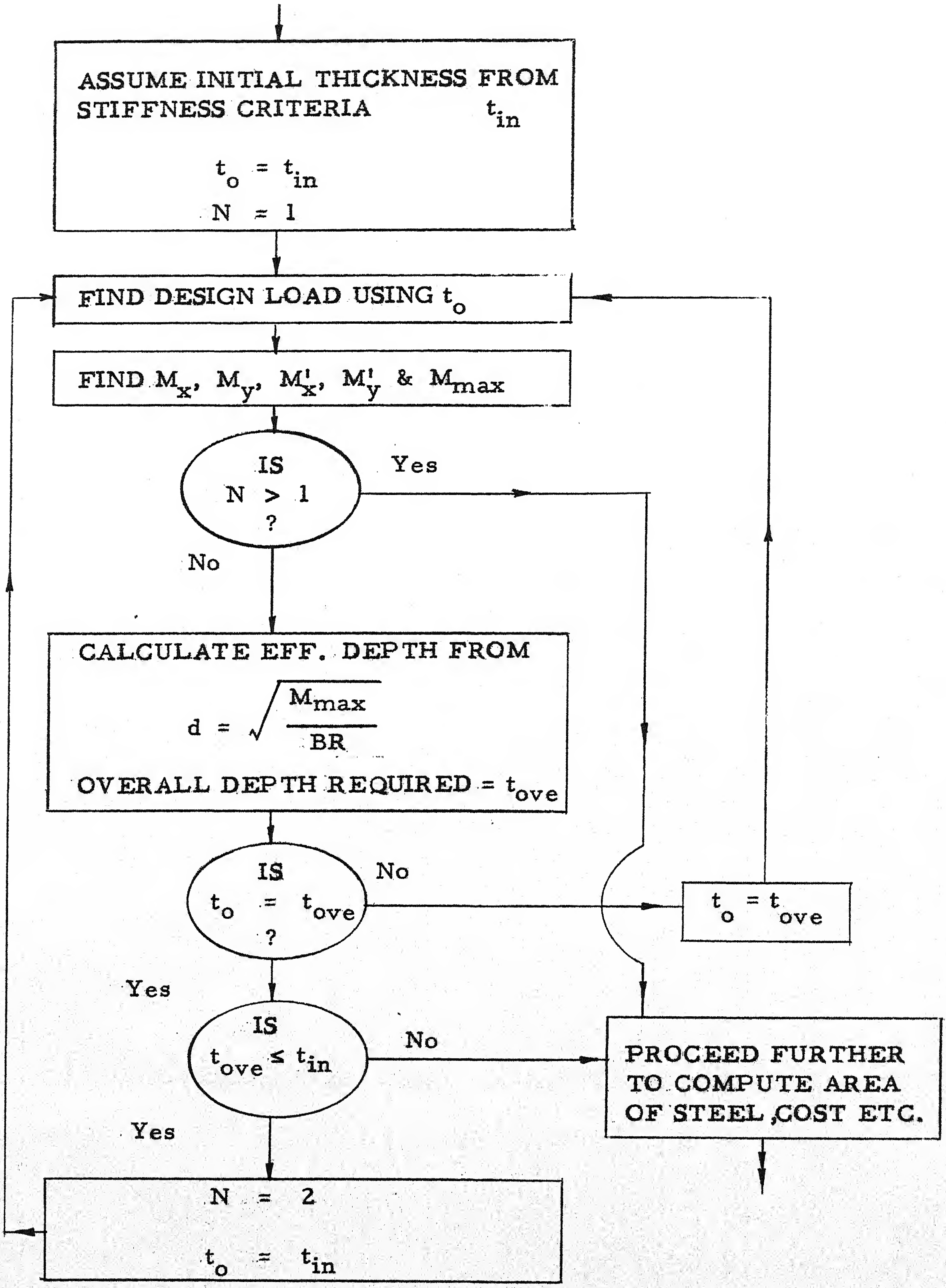


FIG. 5.1: FLOW CHART TO MATCH ASSUMED OVERALL THICKNESS AND COMPUTED OVERALL THICKNESS BY ITERATION

Minimum cost design solution, as evident from Fig. (5.2) and (5.3) is:

Overall slab thickness (t)	....	17.5 cms
$A_{stx} = A_{sty}$	...	10.56 cm^2/m
Total steel per panel = S_w	...	408 Kg.

where

A_{stx} and A_{sty} are areas of reinforcements in x and y directions respectively.

This solution represents the minimum cost solution, as evident also from the following discussion.

If the thickness is increased from 17.5 cms, then the section becomes under reinforced (w.r. to the elastic theory) and the lever arm factor (j) in the formula:

$$A_{st} = \frac{M}{\sigma_{st}\ j\ d} \qquad \ldots \quad (5.12)$$

is also increased. Due to an increase in the lever arm factor (j) and an increase in the overall thickness (t), the amount of steel reduces, but not very much. Cost function (C) is also increased considerably due to the substantial increase in the thickness and the nominal decrease in the amount of steel.

In above Equation (5.12), σ_{st} is the permissible stress in steel in tension and `d´ is the effective depth of the slab in the relevant direction.

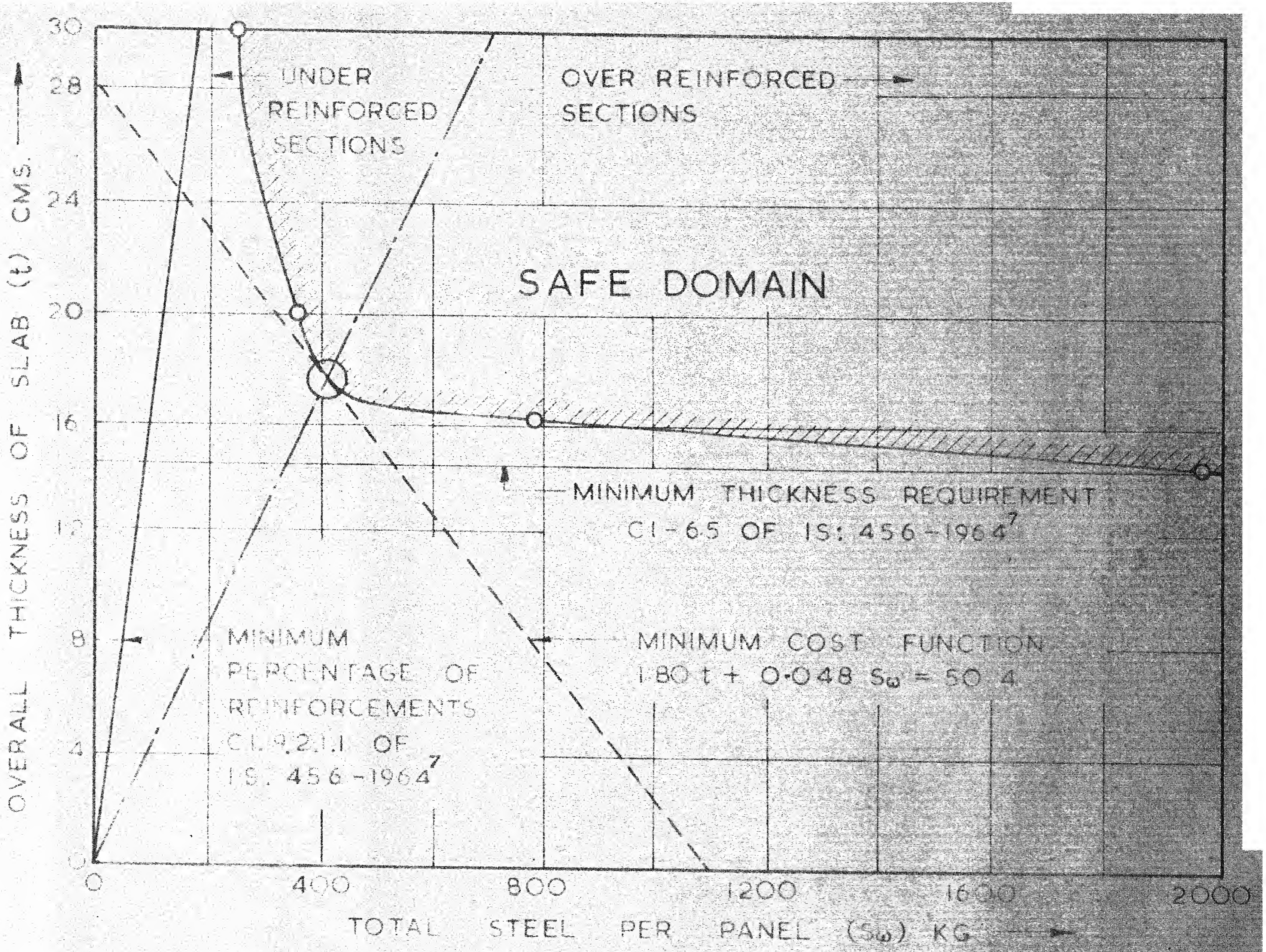


FIG 5.2 MINIMUM COST DESIGN SOLUTION FOR TWO-WAY SIMPLY SUPPORTED SQUARE SLAB

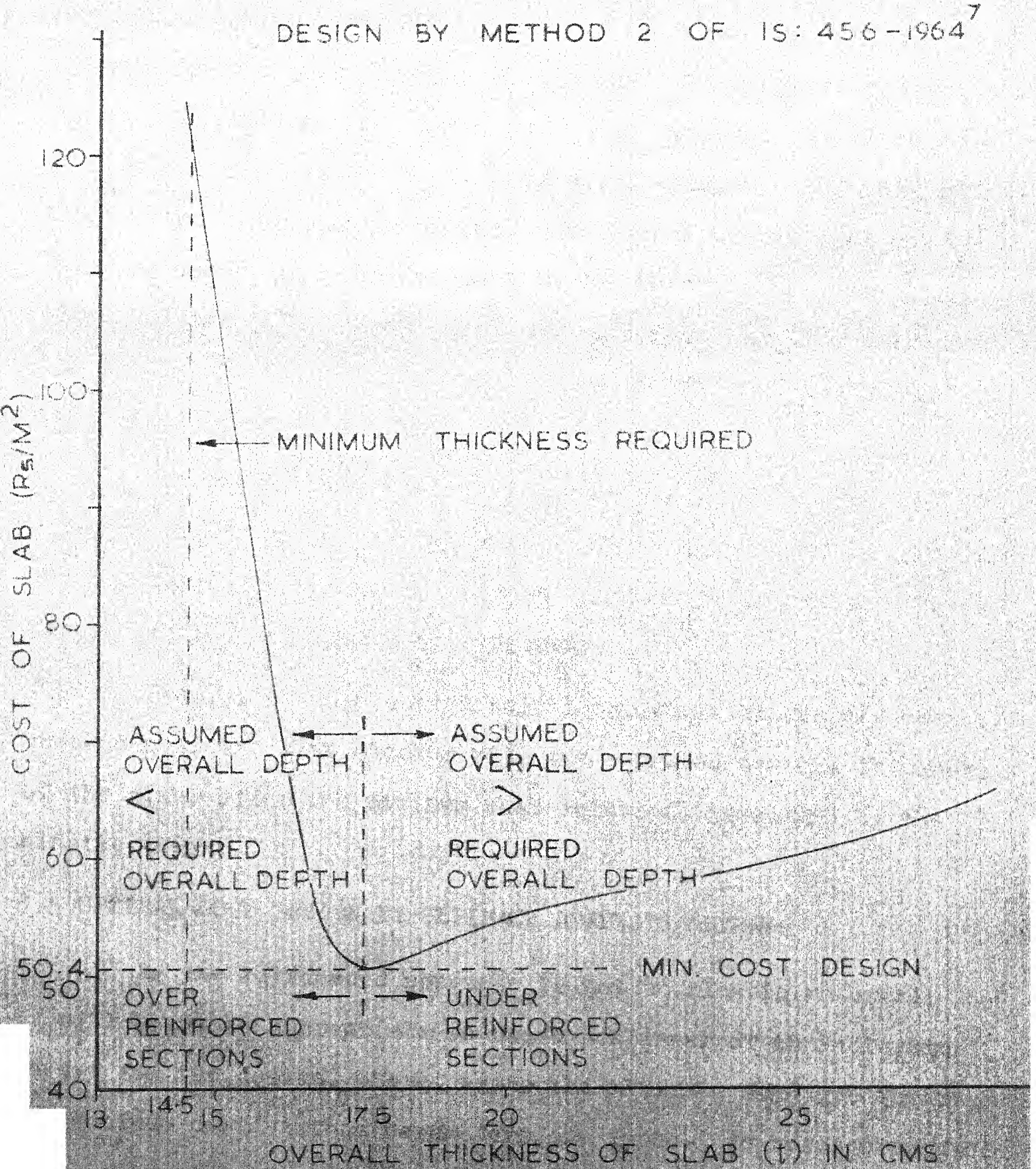


FIG. 5.3 MINIMUM COST DESIGN FROM STUDY OF OVERALL THICKNESS OF THE SLAB AND COST OF SLAB PER UNIT AREA.

Now, if the overall thickness (t), is reduced then the section becomes an over-reinforced one (w.r. to the elastic theory) and the amount of the steel cannot be computed from Equation (5.12), as the actual values of σ_{st} and j will be considerably less than those for a balanced section. So, this will involve the considerable amount of the steel (as it will not be fully stressed) and the decrease in the thickness is very less (Fig. 5.2 and 5.3). Here, also the cost function C is increased due to the substantial increase in the amount of the steel and the nominal decrease in the thickness of slab.

It may be noted that the variations in the unit costs of concrete and steel will not influence the design solution obtained for the case presented, as the cost function represented by Equation (5.11) do not change considerably.

Thus, iterative design solutions, where an assumed overall thickness of the slab matches with the computed overall thickness of the slab, offer the minimum cost solutions, designed by the elastic theory.

5.5 OPTIMAL COST DESIGN BY ULTIMATE STRENGTH THEORY

As the reinforced concrete is not an elastic material, it will be quite appropriate to use a nonlinear or an inelastic theory for the design. Here, using the ultimate strength theory, for minimum cost various parameters are found. Approach presented here is based on Traum's[68] work, where the maximum design moment in the two-way slab is one of the two span moments. Total

reinforcement in the slab is considered to be proportional to the area of reinforcements required corresponding to the maximum bending moment. Thus, the total cost function is just like Equation (5.8).

For IS: 456-1964[7],

$$M_u = \sigma_{cu}\, bd^2\, q' \,(1 - 0.78\, q') \qquad \ldots \quad (5.13)$$

where σ_{cu} is the cube strength at 28 days of concrete (Kg/cm^2)

b, d are width (usually 100 cms) and effective depth of the two-way slab respectively (cm)

M_u is the ultimate moment

$$q' = p \frac{\sigma_{sy}}{\sigma_{cu}} \qquad \ldots \quad (5.14)$$

where p is the percentage of reinforcement

σ_{sy} is the yield stress of the reinforcement (Kg/cm^2)

Maximum value of q' specified by IS:456-1964[7],

$$q'_{max} = p_{max} \frac{\sigma_{sy}}{\sigma_{cu}} = 0.236 \qquad \ldots \quad (5.15)$$

So,

$$p_{max} = 0.236 \frac{\sigma_{cu}}{\sigma_{sy}} \qquad \ldots \quad (5.16)$$

and the cost of the two-way slab per unit area

$$C = \frac{(d+2.5)\ C_c}{100} + 1.05 \times 100 \times d \times 0.785 \times C_s \times \bar{\alpha}(1+\mu)\,p_x \qquad \ldots (5.17)$$

where 2.5 is an average effective cover (Cm)

0.785 is weight of reinforcing steel in Kg/cm^2 per m. length

1.05 is a coefficient to allow lap bands, hooks etc.

C_c is the cost of 1 m^3 of concrete in place (including shuttering)

C_s is the cost of 1 Kg. of reinforcing steel in place

$\bar{\alpha}$ is the coefficient defining the average percentage of reinforcing steel over entire length (approximately = 0.8)

μ is equal to the ratio $M_{uy}/M_{ux} \leq 1$

M_{ux}, M_{uy} are the ultimate moments in X and Y directions respectively.

In Equation (5.17) it is assumed,

$$p_y \simeq p_x \frac{M_{uy}}{M_{ux}} = p_x \qquad \ldots (5.18)$$

i.e. considering effective depths in X and Y directions to be same.

Also $\quad M_{ux} > M_{uy} \qquad \ldots (5.19)$

Using Equation (5.13),

$$d = \sqrt{\frac{M_{ux}}{q' \; 100 \, \sigma_{cu} \; (1-0.78q')}} = \sqrt{\frac{M_{ux}}{100 \; R}} \quad \ldots \quad (5.20)$$

where

$$R = q' \, \sigma_{cu} \quad (1 - 0.78 \, q') \quad \ldots \quad (5.21)$$

Putting Equation (5.20) in Equation (5.7),

$$C = \sqrt{\frac{M_{ux}}{100 \; R}} \left[\frac{C_c}{100} + 1.05x100x0.785xC_s \, \bar{\alpha} \, (1 + \mu) \, p_x \right] + \frac{2.5 \; C_c}{100} \quad \ldots \quad (5.22)$$

To get an ideal percentage of steel reinforcements for minimum cost,

$$\frac{dC}{dp_x} = 0, \quad \ldots \quad (5.23)$$

keeping in mind $R = R(p_x)$

After simplifying the results of Equation (5.23),

$$p_{xopt} = \frac{C_c/100}{105 \text{ x } 0.785 \; \bar{\alpha} \; C_s(1+\mu) + \frac{C_c}{100} \text{ x } 1.56 \frac{\sigma_{sy}}{\sigma_{cu}}} \quad \ldots \quad (5.24)$$

From Equation (5.24), for known values of C_c, C_s, σ_{sy}, σ_{cu} and $\alpha \simeq 0.8$, $p_{x \; opt}$ can be easily found. Then using Equation (5.21) and Equation (5.20), one can get the overall thickness.

5.5.1 Illustrative Example

Given

$$\sigma_{sy} = 2600 \text{ Kg/cm}^2$$

$$M_{ux} = 200{,}000 \text{ Kg.cm}$$

$$= 0.9$$

$$\sigma_{cu} = 200 \text{ Kg/cm}^2 \text{ (M.200 concrete)}$$

$$C_c = \text{Rs. } 180/- \text{ per } M^3$$

$$C_s = \text{Rs. } 1/20 \text{ per Kg.}$$

To find

Overall thickness of slab and the percentage of steel in X and Y direction for the minimum cost by the ultimate strength theory.

Using Equation (5.24)

$$p_{x\ opt} = 0.00965$$

$$q'_{opt} = 0.126 < 0.236 \qquad \text{So, o.k.}$$

R from Equation (5.21) $= 0.126 \times 200\ (1-0.78 \times 0.126)$

$= 22.8$

Then, effective depth (in x direction) (from Equation (5.20)) $= 9.42$ (cms)

Then, the overall thickness of this slab

$= 9.42 + 2.5 = 11.92$ (cms)

or say 12.00 cms.

$$p_{y\ opt} = 0.9 \times 0.00965 = 0.0087$$

5.6 COMMENTS ON OPTIMUM COST DESIGN METHODS

These methods will be quite stressing their economic importance when large number of repetitions are involved in any building construction. For small jobs, they will involve considerable amount of time in the design office which cannot be justified on economic grounds.

In particular, Traum's[68] approach is perfectly correct for the case of simply supported rectangular slabs, but it fairs very badly for continuous two-way and flat slabs. This is due to the domination of support moments over span moments. Thus, for continuous two-way slabs, the approach suggested by Thakkar and Rao[47,67], is quite helpful for both working stress method as well as inelastic theory.

Norman[69] in his paper, draws the so obvious conclusion that for minimum cost, the concrete should be leanest possible. This is already in the practice to use M.150 concrete for the two-way slabs. Neither Norman[69] nor Traum[68] satisfies the minimum thickness constraint suggested by the codes, while the work of Thakkar and Rao[67], do satisfy this requirement.

CHAPTER 6

DESIGN OF COMPLEX SLABS BY HILLERBORG'S APPROXIMATE THEORY OF ELASTICITY

6.1 INTRODUCTION

In this chapter a preliminary discussion of the 'Advanced Strip Method' of Hillerborg for complex slabs is given. When the two-way slab is supported by an intermediate column support or a wall support and/or the supporting boundary is having the re-entrant corners, (Fig. 6.1) the elastic analysis is very complicated and tiresome for an average structural designer. He can solve the problem by either providing the stiff beams or hidden or concealed beams within the slab (a very strong band of reinforcement to act like a beam), to form the layout in which only rectangular panels are considered. But when the aesthetics and sometimes 'headroom' considerations require a clear soffit i.e.without any projecting beams. These cases can be however designed by some approximations in the theory of elasticity.

Hillerborg[17] to overcome the difficulties in the design of such complex slabs, made certain assumptions, in his equilibrium theory, over and above those of 'strip method'. He made use of frame analysis to get the moment field in both the directions with the assumption of zero twisting moment and averaging the total moment field in a strip so that uniform reinforcement

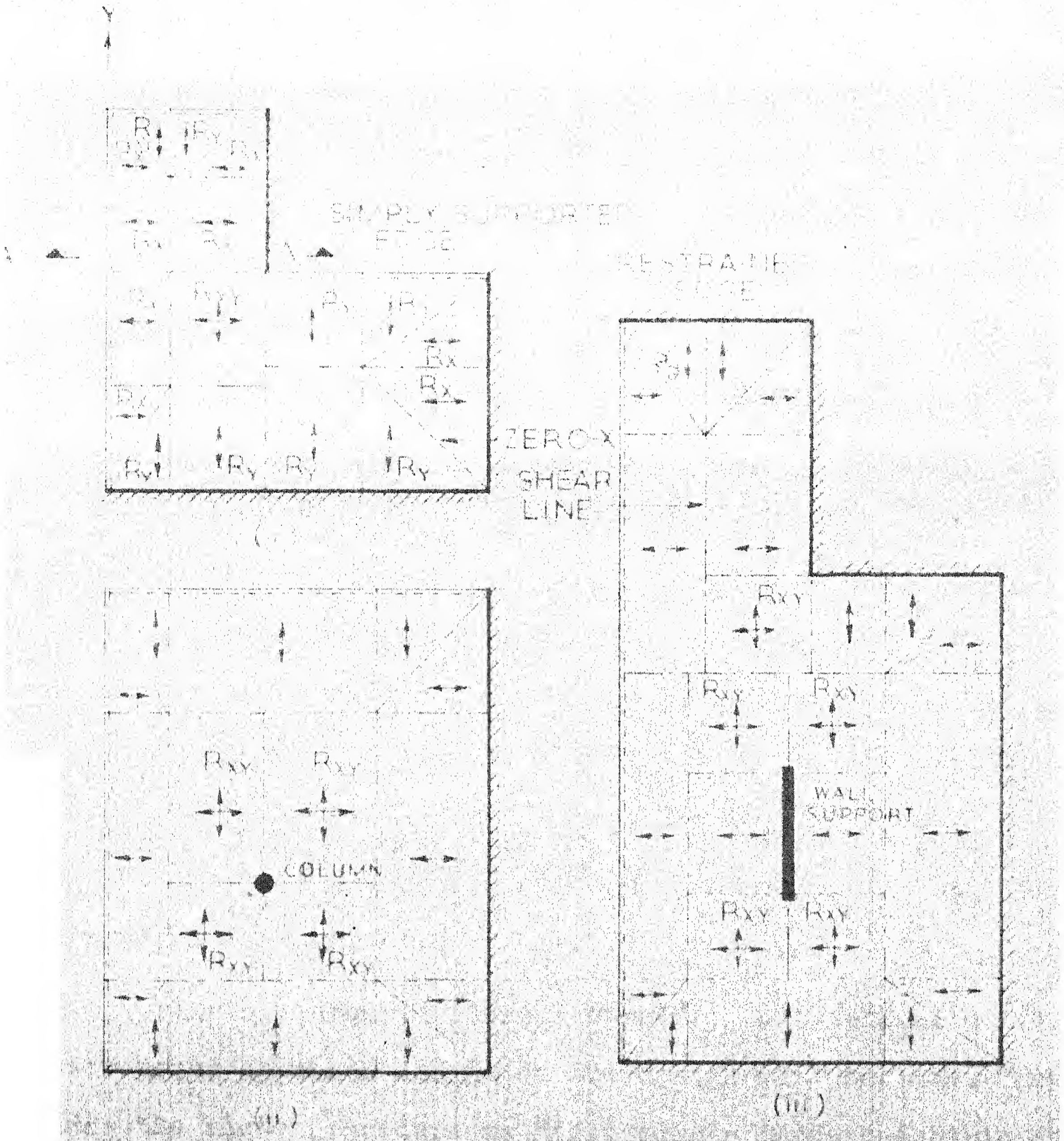


FIG. 6.1 HILERBORG'S STRIPS IN THE SLABS WITH A REENTRANT CORNER INTERMEDIATE COLUMN OR WALL SUPPORT (HILLERBORG[17])

can be placed. Thus, inherently, the assumption of full moment redistribution is used. When the equations of theory of plates is used, the design is with variable reinforcement as discussed in chapter 4. Of course, the redistribution of moments in such cases can also be tried.

6.2 HILLERBORG'S APPROXIMATE THEORY OF ELASTICITY

Using the rules to divide the slabs as discussed already in chapter 3, Hillerborg was confronted with the problem for certain region (a region at the re-entrant corner or column) (Fig. 6.1) for the load distribution in x and y direction.This region can not be considered R_x or R_y. To overcome this difficulty to findout actual load distribution in two directions Hillerborg assumes on very conservative and safe side.

'In the region R_{xy}, the load transferred in both the directions is equal to q'.

Like Hillerborg's Strip Method[16], this will also lead to lower bound approach and hence the design will be safe. Crawford names the above procedure as Hillerborg's 'Advanced Strip Method'. Advanced strip method is a direct design approach where a check for the punching shear at the column or the re-entrant corner is made for the slab thickness. Advanced strip method can be applied for any degree of restraints at boundaries from simply supported to clamped. This is done by considering a ratio:

$$\phi = \frac{M'}{M^F} \quad \ldots \quad (6.1)$$

Where M' = Actual moment at the boundary

and M^F = Fixed-end moment

In advanced strip method, the value of ϕ at the re-entrant corner or column is always assumed to be 0.75.

Denoting,

$$M_s = C_s \, ql^2$$

$$M_f = C_f \, ql^2, \text{ and} \qquad \ldots \qquad (6.2)$$

$$M_p = C_p \, ql^2$$

where M_s = support (other than the re-entrant corner, column or wall) moment (negative)

M_f = span (positive) moment and

M_p = support (negative) moment at the support (the re-entrant corner, column or wall)

C_s, C_f, C_p = moment coefficients corresponding to M_s, M_f and M_p respectively.

q = design load

l = span of the strip

Following relations can be easily derived from Fig.6.2.

$$C_s = \phi_s \left| \frac{1}{8} - \frac{1}{2} C_p \right| \qquad \ldots \qquad (6.3)$$

$$C_p = \phi_p \left| \frac{1}{8} - \frac{1}{2} C_s \right| \qquad \ldots \qquad (6.4)$$

$$C_f = \frac{1}{8} - \frac{(C_s + C_p)}{2} + \frac{(C_s - C_p)^2}{2} \qquad (6.5)$$

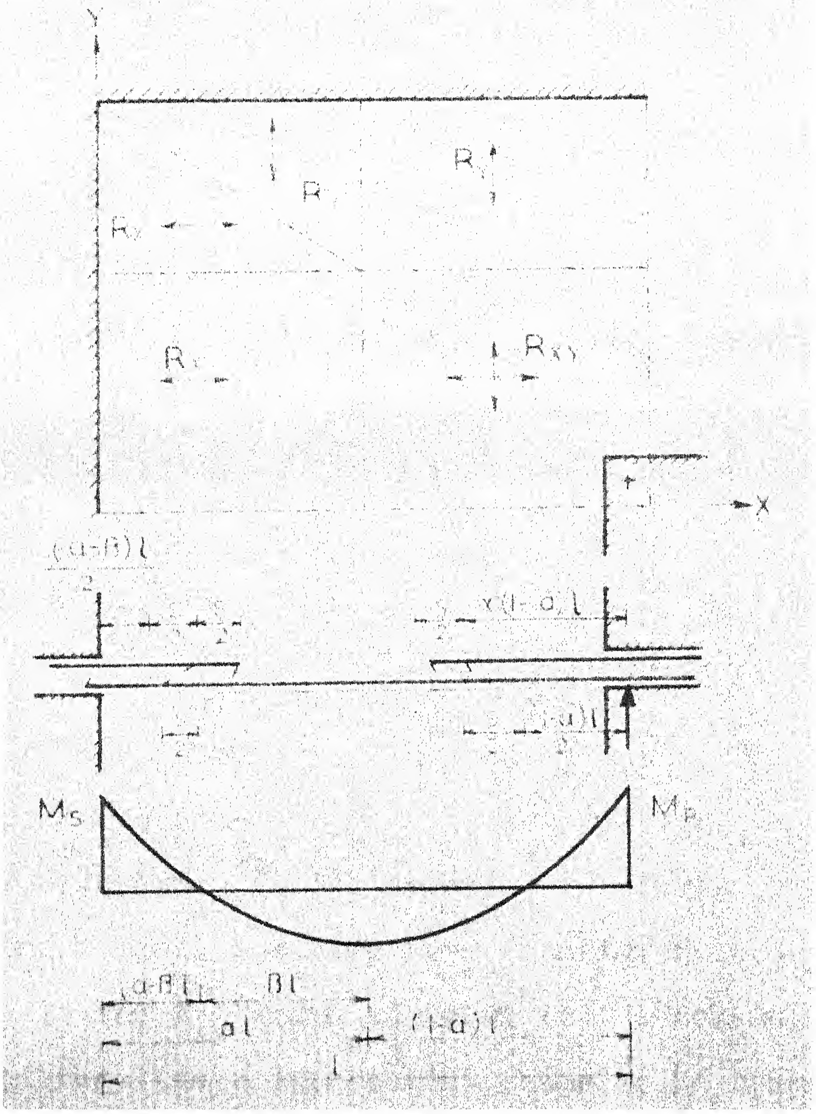


FIG. 6·2 HILLERBORG'S RULES FOR TERMINATION OF REINFORCEMENT (HILLERBORG)[7]

Substituting Equation (6.4) in Equation (6.3) and then solving for C_s,

$$C_s = \frac{\phi_s \left| \frac{1}{8} - \frac{1}{16} \phi_p \right|}{1 - \frac{\phi_p \phi_s}{4}} \quad \ldots \quad (6.6)$$

In Equations (6.3) to (6.6) ϕ_s and ϕ_p are the factors defining the degree of restraint (as in Equation (6.1)) for M_s and M_p respectively.

With the use of Equations (6.6), (6.4) and (6.5) in the order, moment field for the any strip can be had. In advanced strip method, for R_{xy}, ϕ_p is assumed to be 0.75.

For the distribution of reinforcement in the region R_{xy}, positive reinforcement (for M_f) is provided for the moments computed from Equation (6.2). Negative reinforcement is provided for twice the moments computed from Equation (6.2) and distributed over only half the R_{xy} width adjacent to the column.[17] This distribution of negative reinforcement seems to be reasonable, as the negative moments would peak up at the re-entrant corner, column or wall support and then decrease in the magnitude and eventually become positive at some distance from the column. Here, statics in terms of total moments, is satisfied.

To economize the use of reinforcement, Hillerborg[17] suggested the rules for curtailment, which are in the practice in Sweden.[11]

6.3 HILLERBORG'S RULES FOR CURTAILMENT OF REINFORCEMENT

For finding the curtailment of reinforcement the point of contraflexure for equivalent frames by elastic analysis together with bond and anchorage lengths is found. Fig. 6.2 shows Hillerborg's[17] rules for curtailment of reinforcement α and β are coefficient defined in following equations,

$$\alpha = 0.5 + c_s - c_p \qquad \ldots \qquad (6.7)$$

$$\beta = \sqrt{2 c_f} \qquad \ldots \qquad (6.8)$$

and γ is found out from Fig. 6.3 using values of c_f and c_p found out from Equations(6.4) and (6.5). S is the anchorage length of reinforcement, but not less three times the effective depth of the slab.

6.3.1 Illustrative Example

In Fig. 6.1 (i), for strip AA, it is required to find out moment field and the curtailment of reinforcement.

Here, $\phi_s = 0;\ \phi_p = 0.75$

From Equation (6.6) $\quad C_s = 0$

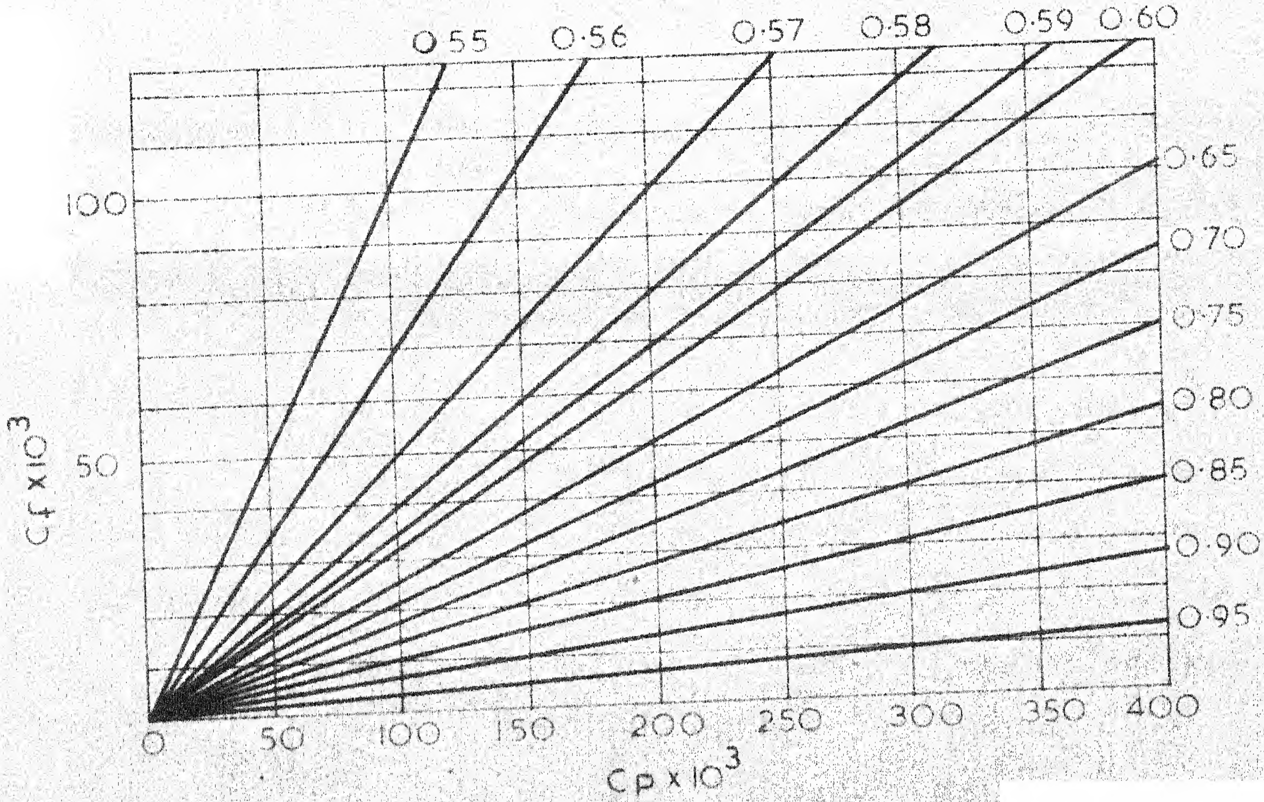
Equation (6.4) $\quad C_p = 0.75 \times \frac{1}{8} = \frac{3}{32} = 0.094,$

and Equation (6.5) $\quad C_f = \frac{1}{8} - \frac{3}{64} + \frac{9}{32 \times 32} \times \frac{1}{2} = 0.082$

for curtailment of reinforcement,

From Equation (6.7) $\quad \alpha = 0.5 - 0.094 = 0.406$

Equation (6.8) $\quad \beta = \sqrt{2 \times 0.082} = 0.406$



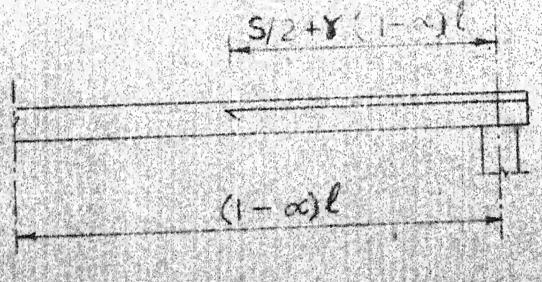


FIG. 6·3 LENGTH OF NEGATIVE REINFORCEMENT AT A REENTRANT CORNER OR A COLUMN (HILLERBORG)[17]

and from Fig. 6.3 for $C_f = 0.082$, $C_p = 0.094$

$$\gamma = 0.555$$

In the next chapter (section 7.4) illustrative design examples will be studied in detail to show merits of the method for design of the complex slabs.

Hillerborg[17] has given a detailed discussion of the limitations of his approach together with applications for various cases. A thorough treatment of the link between Hillerborg's method and the theory of elastic plates with suitable checks on its plastic limit capacity and serviceability restrictions at working loads, is recommended for further research especially for the problem of flat slabs, two-way slabs on flexible beams supported on columns and complex boundary conditions.

CHAPTER 7

ILLUSTRATIVE DESIGN EXAMPLES

7.1 GENERAL REQUIREMENTS AND DATA FOR THE DESIGN EXAMPLES

The following general requirements are satisfied for the relavent design example (or otherwise specified in a particular example):

1. Analysis is done by MHSM (or Hillerborg's approximate theory of elasticity in the case of two-way slabs supported by an re-entrant corners etc.).
2. Design for determining the slab thickness and reinforcement is done according to the 'working stress design' method of IS: 456-1964[7].
3. Minimum thickness of the slab, minimum percentage of reinforcement, minimum cover, maximum spacing of bars and minimum anchorage length required beyond the theoretical point of cut-off; are satisfied using relavent clauses of IS: 456-1964[7].
4. Slabs are checked for the purching shear, if any.
5. Reinforcement cut-off is done using Hillerborg's recommendations (as discussed chapter 6) in confirmation with the relavent clauses of IS: 456-1964[7].

Material properties and permissible stresses according to IS: 456-1964[7] are, used in the design examples as per the

1. Concrete

M.150 (σ_{cu} = 150 Kg/cm^2)

σ_{cb} = 50 Kg/cm^2

τ_a = 5 Kg/cm^2

2. Reinforcement

Mild steel grade I plain bars in confirmation with IS: 432-1960.

σ_{sy} = 2600 Kg/cm^2

σ_{st} = 1400 Kg/cm^2

For the better comparison, design solutions by other known methods were worked out to focuss the economy affected. Here, the costs are worked out on the following assumptions, at the present rate in India:

1. The cost of concrete including shuttering etc., is assumed to be Rs. 180/- per m^3 for M.150 concrete.
2. The cost of mild steel including fabricating, binding, placing in position, wastage, rolling margin etc. is assumed to be Rs. 1/20 per Kg. (Mild Steel Grade I)

Wherever possible, the reinforcement layouts are provided to illustrate the use of MHSM, optimal plastic design etc.

7.2 SIMPLY SUPPORTED RECTANGULAR SLAB BY MHSM

It is required to design a simply supported slab of 3 m by 4.5 m for the live load of 400 Kg/m^2.

$$l_x = 3^m$$
$$l_y = 4.5^m$$
$$\frac{l_y}{l_x} = 1.5$$

Design load

400 Kg/m^2	Live load
240 Kg/m^2	Self weight (Assume 10 cm thick slab
110 Kg/m^2	Finishing (5 cms)
$q =$ 750 Kg/m^2	

By Modified Strip Method (Table 3):

For s = 1.5, from Table 3, $m_x = 14.4$ and $m_y = 54.0$

So $$M_x = \frac{750 \times 3^2}{14.4} = 469 \text{ Kg-m/m}$$

and $$M_y = \frac{750 \times 4.5^2}{54.0} = 282 \text{ Kg-m/m}$$

For working stress design $\sigma_{cb} = 50$ Kg/cm^2;

$\sigma_{st} = 1400$ Kg/cm^2 and $R = 8.67$.

Effective depth $d = \sqrt{\frac{469}{8.67}} = 7.4$ cm

Limiting stiffness criteria $\frac{L}{t} = 35$,

where L = Shorter of the two spans = 300 cms

t = Overall depth of the slab

Choose t = 10 cm with 1.5 cm clear cover,

So, A_{st} in the x direction (3 m span) $= \dfrac{469}{1400x.87x8.1}$

$= 4.87\ cm^2/m$

Provide 12 mm Ø at 23 cm c/c

So, Actual A_{st} in y direction $= 4.92\ cm^2/m$

Similarly A_{st} in y direction $= \dfrac{282}{1400x0.87x6.9} = 3.36\ cm^2/m$

Provide 10 mm Ø at 23 cm c/c

So, Actual A_{st} in y direction $= 3.42\ cm^2/m$

As a check for finding the factor of safety, ultimate analysis by the yield line theory is made. Plastic moment as per Appendix B of IS: 456-1964[7]

in x direction = 950 Kg-m/m

and in y direction = 567 Kg-m/m

(The average lever arm factor at ultimate is 0.915 as compared with 0.87 for working stress design).

Neglecting the corner mechanism, from yield line analysis $q_u = 1530\ Kg/m^2$, and the factor of safety is 2.05. However using corner mechanisms to get the least value of ultimate load intensity q_u, is found to be $1450\ Kg/m^2$ with a factor of safety of 1.94. This is slightly greater than the ratio of yield stress to its maximum permissible stress i.e. 1.86.

7.2.1 Comparative Study

A comparative study for this example of the modified strip method and Methods 2 and 3 of IS: 456-1964[7] without torsion reinforcement is given in Table 10. Method 3 is found to be more conservative than Method 2. Also, it is seen that the savings in steel reinforcement by the use of modified strip method is 10 percent, and the total savings in cost is 7 percent as compared with Method 2.

Table 10: COMPARATIVE STUDY OF SOLUTIONS OBTAINED BY VARIOUS DESIGN PROCEDURES FOR THE SIMPLY SUPPORTED RECTANGULAR SLAB OF 3^m x 4.5^m

Design Method	IS:456-1964 Appendix C Method 3 WSD	IS:456-1964 Appendix C Method 2 Without torsion reinforcement WSD	MHSM-Slab thickness and Design by WSD IS:456-1964
Overall Slab Thickness cms	10.0	10.5	10.0
Factor of safety	2.25	2.13	1.94
Concrete m^3	1.35	1.42	1.35
Steel Kg	175	107	95
Total cost Rs.	452	384	356

7.3 RECTANGULAR SLAB WITH TWO OPPOSITE EDGES CONTINUOUS, BY MHSM

It is required to design the slab shown in Fig. 7.1 with the effective span in x direction (l_x) as 6.85^m and in y direction (l_y) as 5.00^m, for the live load of 200 Kg/m^2

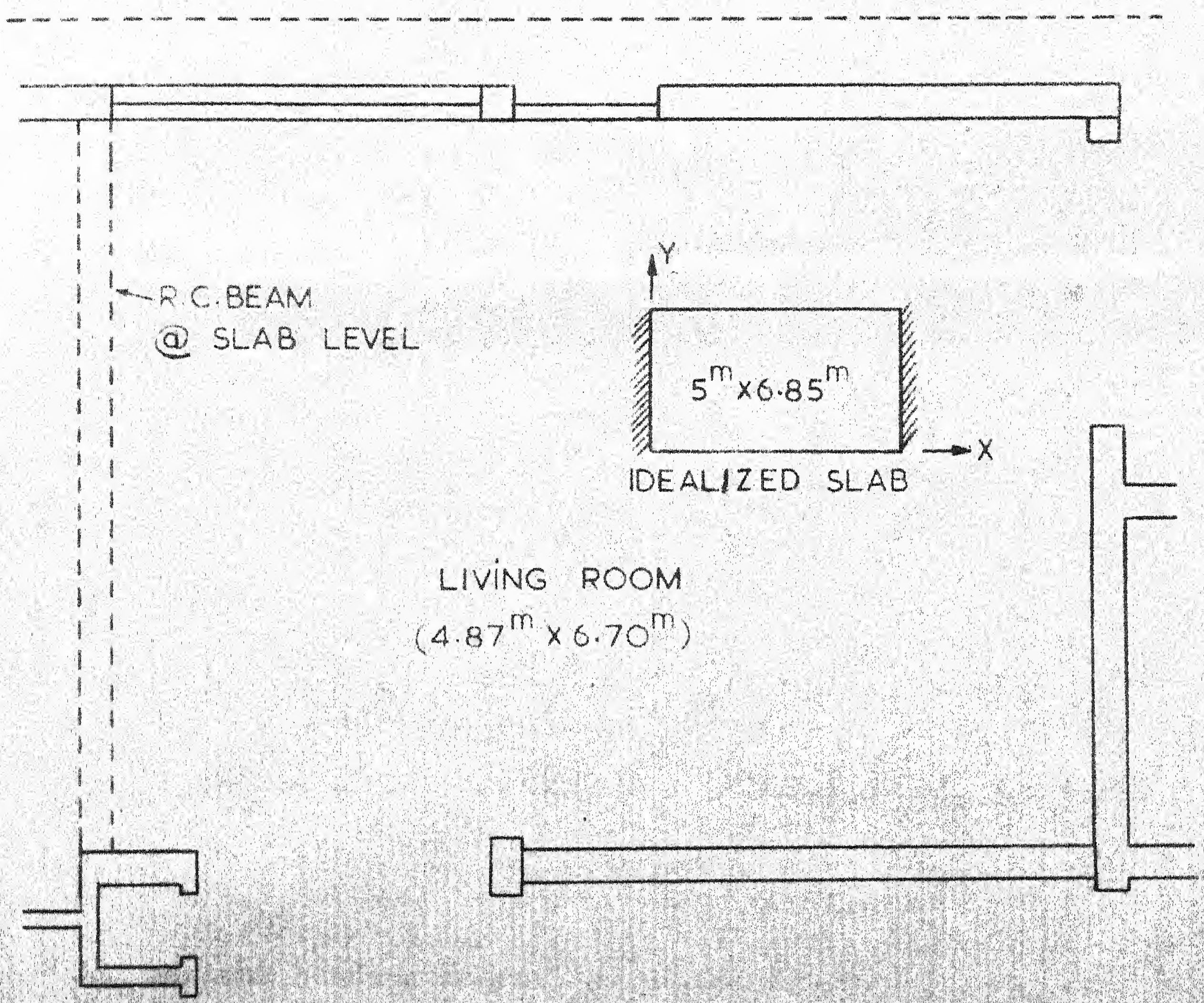


FIG. 7·1 LIVING ROOM DETAILS OF TYPE V QUARTERS IIT/K CAMPUS

$$s = \frac{l_y}{l_x} = 0.73$$

Design Load

200 Kg/m^2	Live load
312 Kg/m^2	Self weight (Assume 13 cm thick slab)
90 Kg/m^2	Finishing
q = 602 Kg/m^2	

Using Equations (3.22)

$$M_x = \frac{ql_x^2}{108} \frac{(4.5s-2)}{s} = \frac{602 \times 6.85^2 \times (4.5 \times 0.73 - 2)}{108 \times 0.73} \times 100$$

$$= 45,800 \text{ Kg-cm}$$

$$M_y = \frac{ql_y^2}{54s^2} = \frac{602 \times 25 \times 100}{54 \times 0.532} = 52,500 \text{ Kg-cm}$$

$$M'_x = \frac{ql_x^2}{27} \frac{(2.25\,s-1)}{s} = \frac{602 \times 6.85^2 \times 0.64 \times 100}{27 \times 0.73}$$

$$= 92,000 \text{ Kg-cm}$$

So, maximum bending moment = 92,000 Kg-cm.

Then effective depth required (using WSD)

$$= \sqrt{\frac{92,000}{100 \times 8.67}} = 10.4 \text{ cms}$$

Provide 13 cms (overall) thick slab with 2 cms clear cover. Here, the overall thickness assumed matches with the required overall thickness, and hence, within the domain of WSD, this design offers an economical solution as discussed in chapter 5.

Mimimum thickness required from the stiffness constraint by clause 6.5 of IS: 456-1964 is 12.5 cms. So, this design with 13 cms as an overall thickness, satisfies the requirement of the code, for the minimum thickness.

Area of steel required in x and y direction has been computed as follows:

$$A_{stx} = \frac{45,800}{1400 \times 0.87 \times 9.5} = 3.96 \ \text{cm}^2/\text{m}$$

Provide 10 mm Ø bars at 20 cms c/c alternate bent-up at supports

$$A_{sty} = \frac{52,500}{1400 \times 0.87 \times 10.5} = 4.10 \ \text{cm}^2/\text{m}$$

Provide 10 mm Ø bars at 19 cms c/c alternate bent-up at supports; and

$$A_{stpx} = \frac{92,000}{1400 \times 0.87 \times 10.5} = 7.20 \ \text{cm}^2/\text{m}$$

Provide 12 mm Ø bars at 20 cms c/c at top as extras at supports.

7.3.1 Reinforcement Layout

Using the rules specified in chapter 3, for the curtailment of reinforcement, the reinforcement layout is shown in Fig. 7.2. It may be noted here that in y direction the bottom reinforcement is bent-up at supports, alternately to cater for the partial continuity, which is ignored in the idealized design problem. This reinforcement layout satisfies clauses 9.2.1, 18.3.3, 18.2 and 18.4.3 of IS: 456-1964.

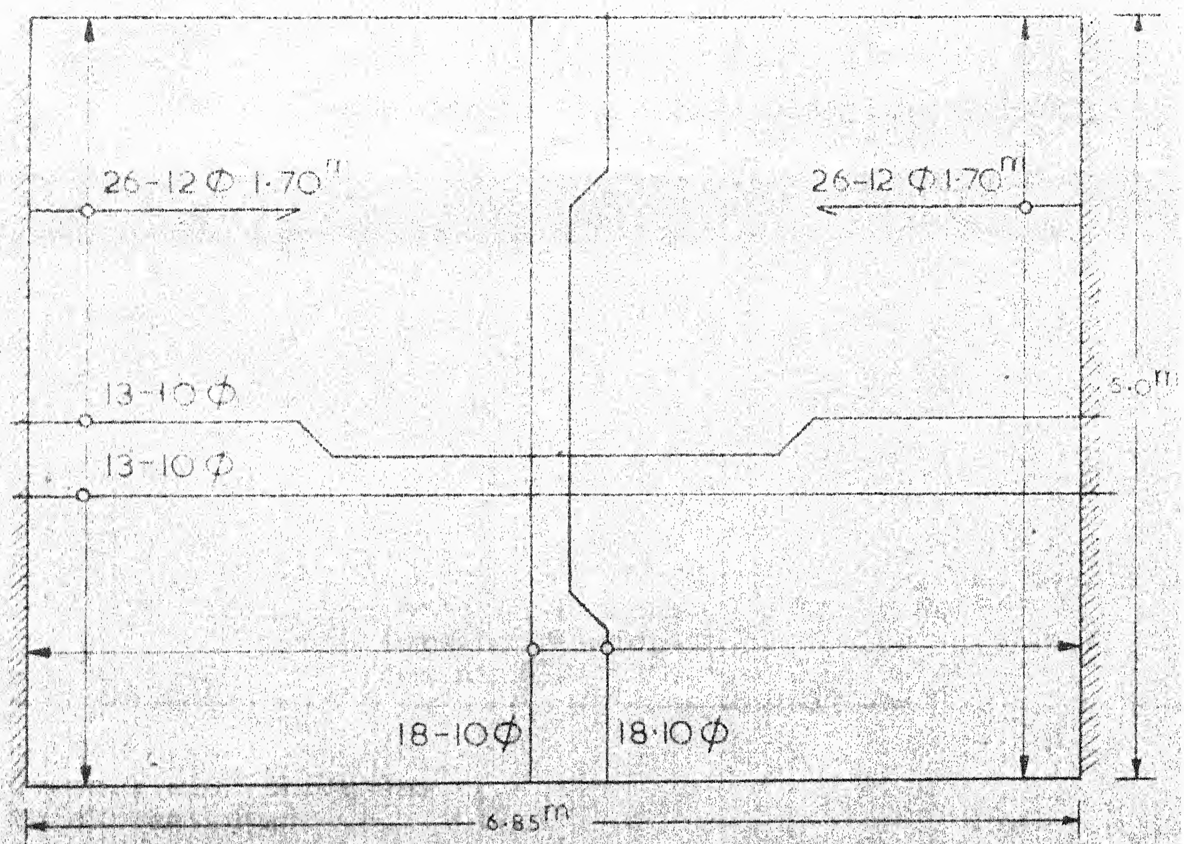


FIG. 7·2 REINFORCEMENT LAYOUT FOR EXAMPLE 7·3

7.3.2 Comparative Study

It can be seen from Table 11, that the design according to the code is involving approximately 23 percent more in cost. This is due to neglect of redistribution of forces in the code provisions.

Effect on economy of the design procedure selection is quite significant for the slab with fixity or continuity on part or whole of the boundary.

Table 11: COMPARATIVE STUDY OF SOLUTIONS OBTAINED BY VARIOUS DESIGN PROCEDURES FOR THE SLAB WITH TWO OPPOSITE EDGES CONTINUOUS (5^m x 6.85^m)

Design Method	IS: 456-1964 Appendix C Method 2 Without torsion reinforcement WSD	MHSM slab thickness and design by WSD IS:456-1964.
Overall slab thickness thickness (cms)	17.0	13.0
Factor of safety	2.55	1.96
Concrete (m^3)	5.82	4.45
Steel (Kg)	328.	303.
Total cost (Rs.)	1444	1163
Cost/m^2 (Rs.)	42.0	34.0

7.4 L - SHAPED SLAB BY HILLERBORG'S APPROXIMATE THEORY OF ELASTICITY

For the live load of 150 Kg/m^2 i.e. for the terrace, design of slab without any beams is presented here for the slab of the room shown in Fig. 7.3.

Design Load:

	150 Kg/m^2	Live load
	408 Kg/m^2	Self weight (Assume 17 cms overall thickness)
	90 Kg/m^2	Finishing
q =	648 Kg/m^2	

Analysis:

Analysis is done in a simple form of design table as shown in Table 12(Col. 1 to 7) using the formulae presented in chapter 6, after the dividing the slab in strips as shown in Fig. 7.4.

Design:

From col. 7, the maximum moment is found to be 1900 Kg-m.

Then, using WSD, the effective depth required

$$= \sqrt{\frac{1900 \times 100}{100 \times 8.67}} = 14.7 \text{ cms}$$

Provide 17.5 cms thick slab with 2 cms. clear cover.

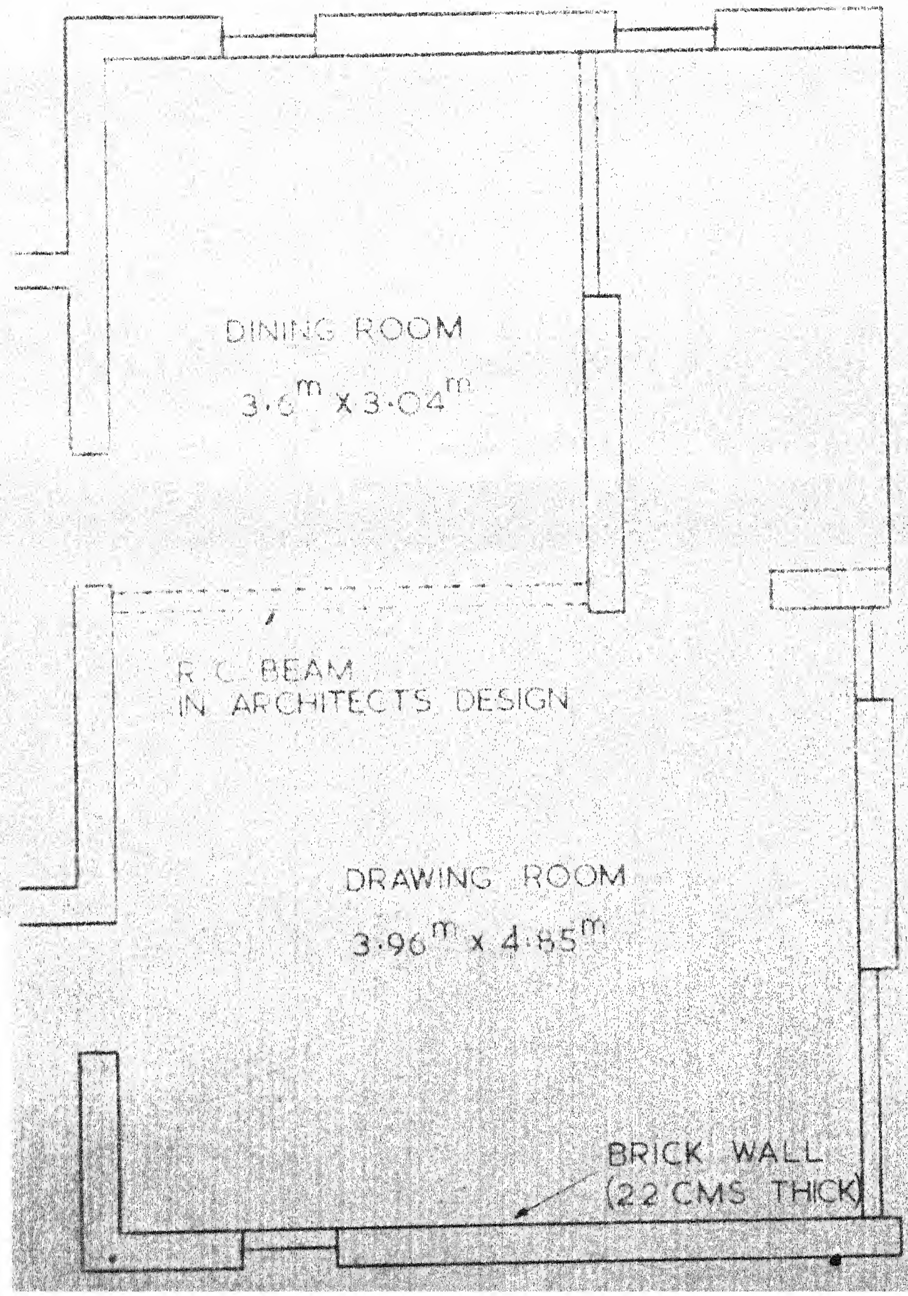


FIG. 7·3 L SHAPED SLAB FOR TYPE IV QUARTERS
IIT/K CAMPUS.

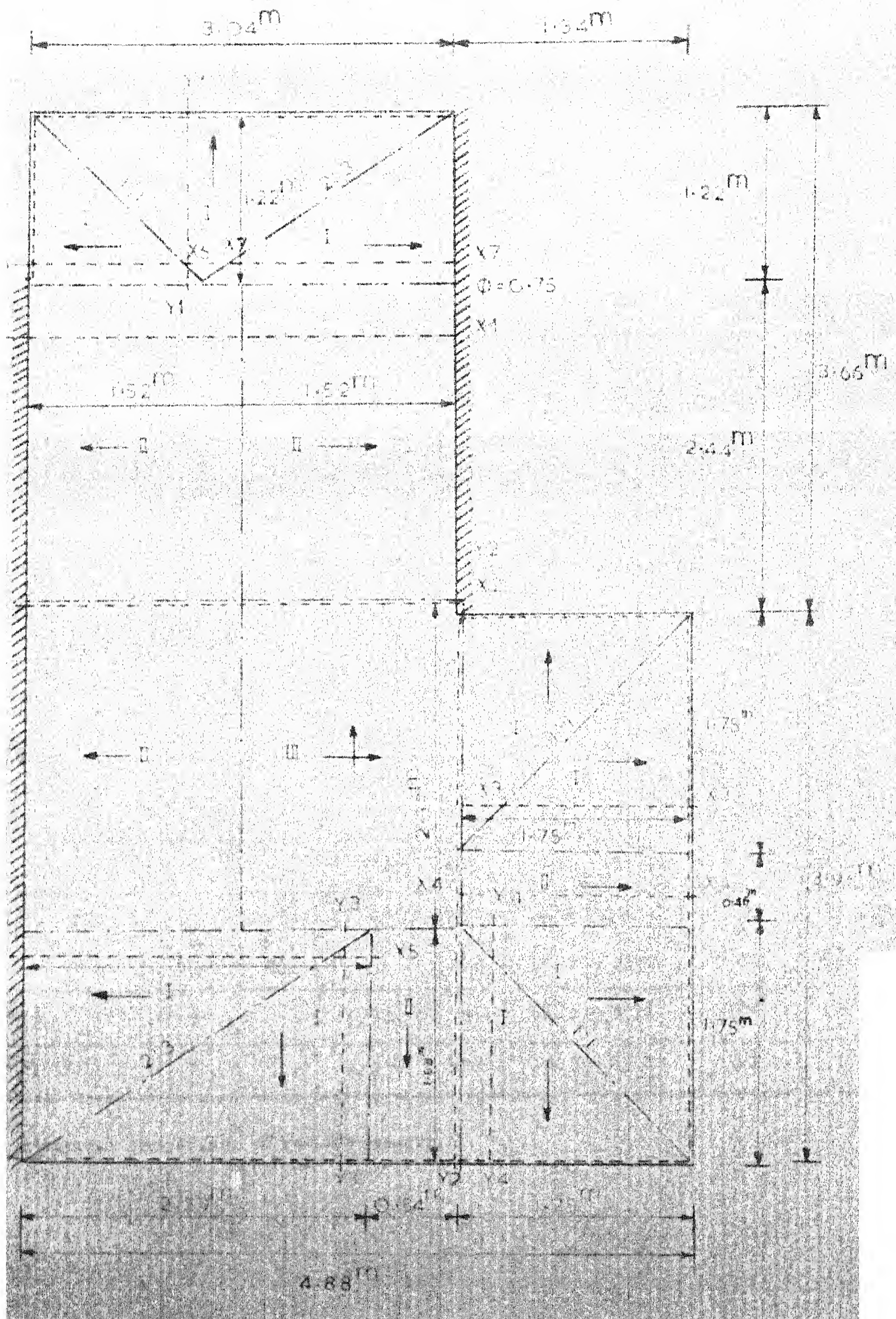


FIG. 7.4 HILLERBORG'S STRIPS IN THE L-SHAPED SLAB.

TABLE: 12 DESIGN TABLE FOR L-SHAPED SLAB $q = 648\ kg/m^2$

Strip	Strip span m	Detail	Index	$Cx10^3$	ql^2 kg-m	Moment kg-m	Eff. depth cms	Area of steel cm^2/m	Dia. of bar mm	Pitch C/C cms	α β γ	$\alpha 1$ $(\alpha - \beta)1$ $(1-\alpha)\gamma 1$ m
1	2	3	4	5	6	7	8	9	10	11	12	13
X 1	3.04	$\phi_s = \phi_p = 0.75$	p s f	68. 68. 57.	6,000	408 408 342	14.9 14.9 14.9	(2.24) * (2.24) (1.88)	10 10 10	31 31 31	.50 .34 .56	1.52 0.48 0.85
X 2	3.04	$\phi_s = \phi_p = 0.75$	2p s f	2x68. 68. 57.	6,000	815 408 342	14.3 14.9 14.9	4.70 (2.24) (1.88)	10 10 10	16.5 31 31	.50 .34 .56	1.52 0.48 0.85
X 3	C=1.75	$M_f = qC^2/2$	f		1,990	995	14.9	5.46	10	14.5		
X 4	C=1.75	$M_f = qC^2/2$	f		1,990	995	14.9	5.46	10	14.5		
X 5	C=2.59	$M_s = qC^2/3$ $M_f = qC^2/6$	s f		4,350	1450 725	14.3 14.9	8.35 4.87	12 10	13.5 16.	1.0 .58	2.59 1.52
X 6	C=1.22	$M_f=qC^2/2$	f		965	482	14.9	(2.61)	10	31		
X 7	C=1.82	$M_s=qC^2/3$ $M_f=qC^2/6$	s f		2,140	713 356	14.9 14.9	3.92 (1.96)	10 10	20 31	1.0 .58	1.82 1.06
Y 1	C=1.22	$M_f = qC^2/2$	f		965	482	14.9	(2.61)	10	31		
Y 2	3.96	$\phi_s=0,\ \phi_p = .75$	2p s f	2x94. 0 40.	10,100	1900 404	14.9 14.3	10.65 (2.32)	12 10.	10.5 31	.41 .41 .57	1.62 0 1.34
Y 3	C=1.68	$M_f = qC^2/2$	f		1,830	915	14.3	5.25	10	15		
Y 4	C=1.75	$M_f = qC^2/2$	f		1,990	995	14.3	5.71	10	14		

Note: * Governed by minimum percentage of reinforcement.

Check for the punching shear
Approx. eff. depth = 15.0 cms

Now, using Hillerborg's rules for checking the depth for the purching shear,

$$P_{critical} = 648 \times 4 \times 1.52 \times 2.37 = 9350 \text{ Kg}$$

$$\tau = \frac{1.15 \times 9350}{15\ (6 \times 23 + 4 \times 15)} = 3.86 \text{ Kg/cm}^2$$

which is $< 0.8 \times 5$ Kg/cm^2, so o.k.

Minimum percentage of reinforcement:

According to the clause 9.2.1 of IS:456-1964, this is

$$\frac{0.15}{100} \times 100 \times 17.5 = 2.62 \text{ cm}^2/\text{m}$$

Reinforcement Design:

Table 12 (col. 8 to 11) gives the reinforcement design. It should be noted that the values in the bracket in the col.9 indicate that these are governed by the minimum percentage of reinforcement, computed above.

Curtailment of reinforcement:

Using 10 mm Ø bars, anchorage length required in the confirmation of the clause 18.4.3 of IS:456-1964 is 48 cms.

So, $S = 48$ cms

Then, using Hillerborg's rules presented in the chapter 6, values of α, β and have been computed and tabulated (for each strip) in Table 12, column 12.

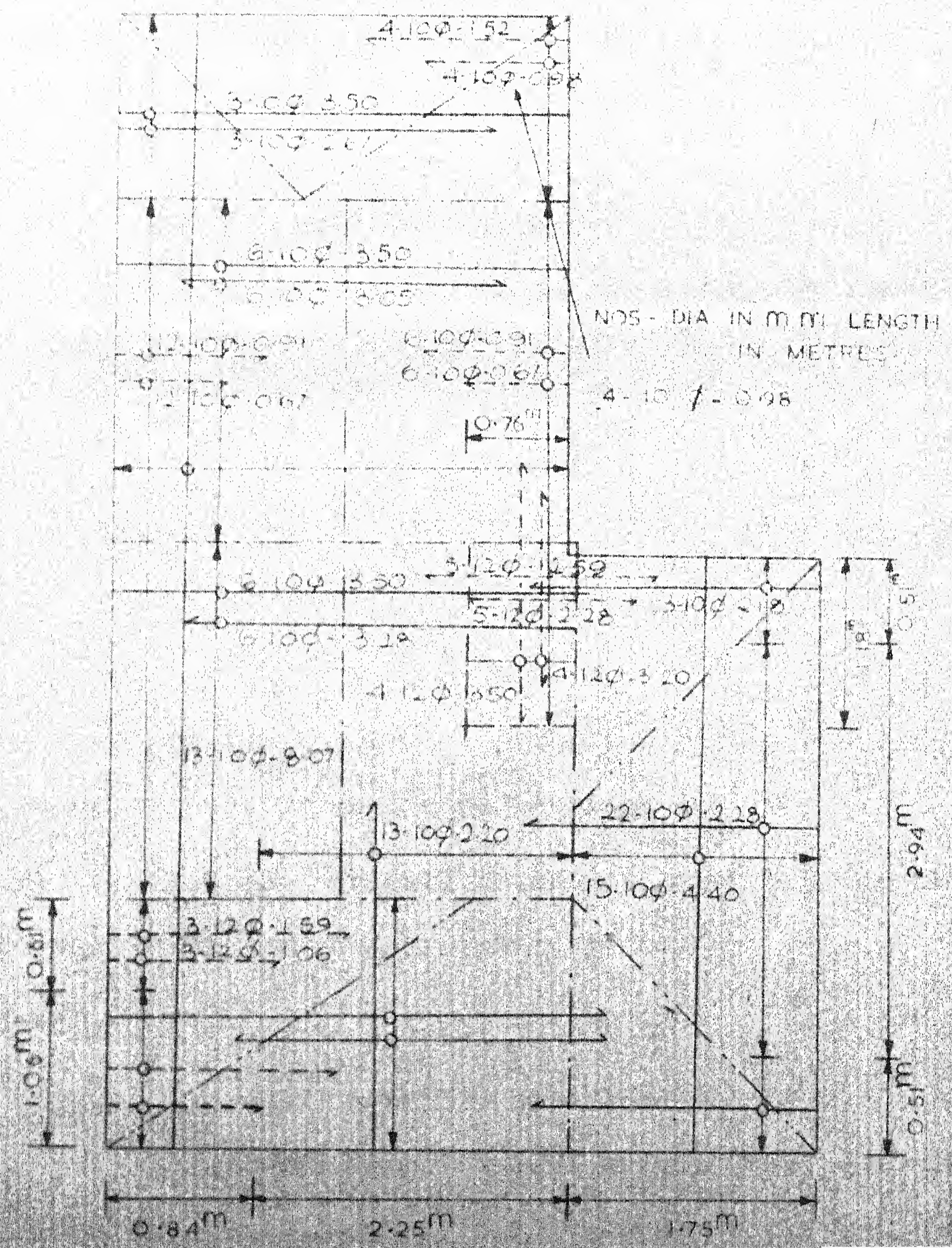


FIG. 7.5 REINFORCEMENT LAYOUT FOR THE L - SHAPED SLAB.

7.5 OPTIMUM PLASTIC DESIGN AS APPLIED TO THE CLAMPED SQUARE SLABS

For the live load of 200 Kg/m^2, optimum plastic design for the clamped square slab of $5^m \times 5^m$ is presented here for both the 'curtailed' and 'uncurtailed' solutions. Here, the clauses related to the minimum percentage of reinforcement and maximum spacing of bars, are waived as already discussed in chapter 4.

From section 4.4.4 the maximum bending moment is equal to

$\frac{3}{32} \; q \, L^2$ (at the centre of the supporting edges)

Slab Thickness:

Here, the slab is provided with 12.5 cms which is determined from the minimum thickness criterion of the code, for safe-guarding against excessive deflection.

Provide 12.5 cms. overall thick slab with 1.5 cms clear cover and use 10 mm Ø bars as reinforcement.

Design Load:

	200 Kg/m^2	Live load
	300 Kg/m^2	Self weight (12.5 cms overall thickness)
	100 Kg/m^2	Finishing
q =	600 Kg/m^2	

$A_{st\ max}$ (at the centre of supporting edges)

$$= \frac{3}{32} \times \frac{600 \times 25 \times 100}{1400 \times 0.87 \times 10.5} = 11.0 \text{ cm}^2/\text{m}$$

Using 10 mm Ø, spacing required will be

$$\frac{100 \times 0.7854}{11.0} = 7.15 \text{ cms. (say 7 cm c/c)}$$

Bending moments at x = L/4 (Fig. 4.7) at the support and the centre of slab are equal to

$$\frac{qL^2}{32}$$

Thus, at these two places the spacings should be 21.35 cms (say 21.00 cm c/c).

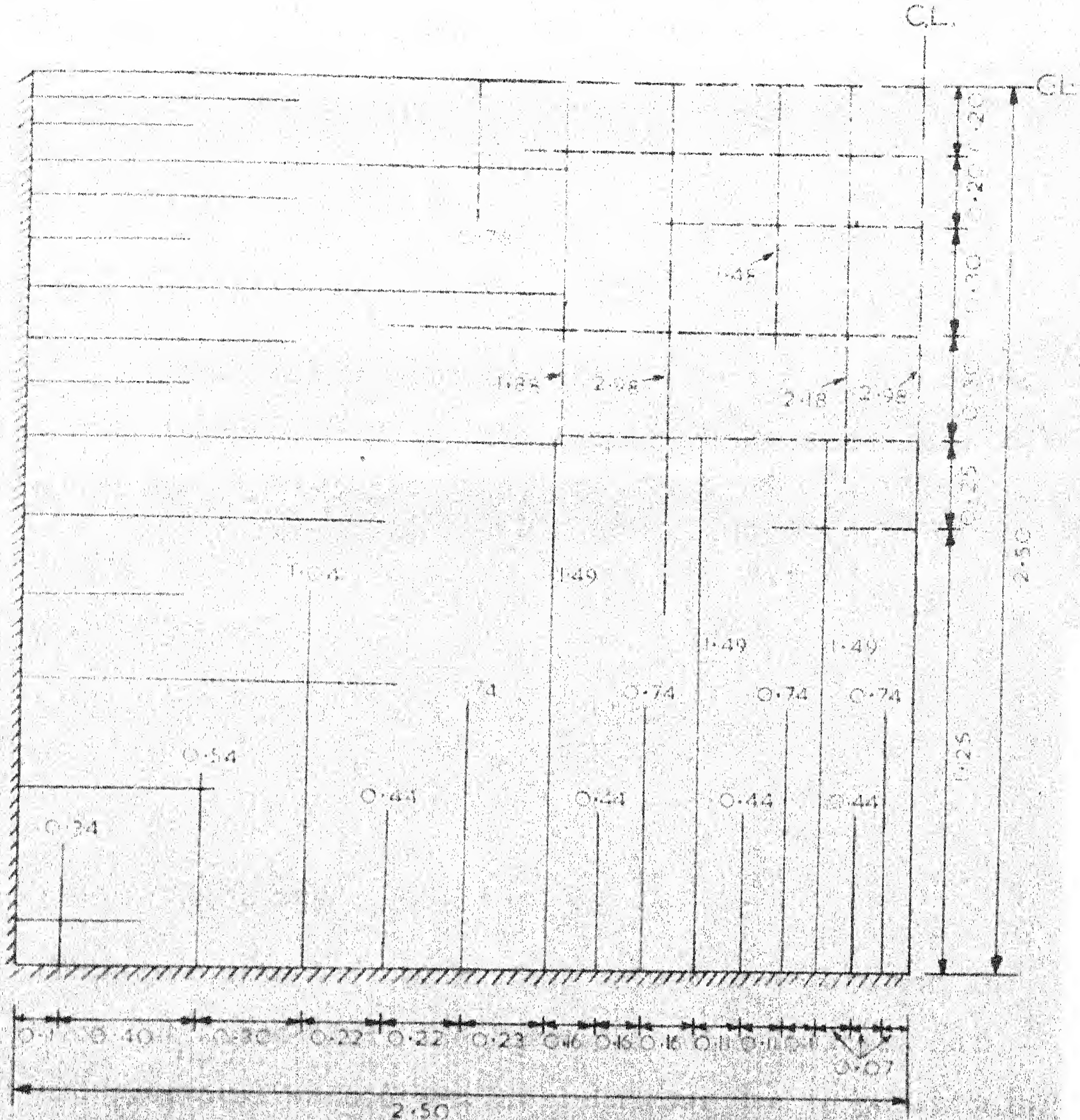
Providing 24 cms more length of bars beyond the theoretical point of cut off for reinforcement, the layouts for the 'curtailed' and 'uncurtailed' solutions can be prepared.

7.5.1 Reinforcement layout for the 'curtailed' solution

Considering the variations both in the spacing and the length of bars for the finite width of slab, a practical layout is prepared as shown in Figure 7.6.

Total length of reinforcement of 10 mm Ø bars from Fig. 7.6 can be found as 109.16 metres.

Value of 'c' (as defined in section 4.4.4) can now be worked out for better comparison with the theoretical one.



(ALL DIMENSIONS IN METRES)

——— FOR SUPPORT MOMENTS, - - - - - FOR SPAN MOMENTS

FIG. 7.6 CURTAILED REINFORCEMENT LAYOUT FOR THE CLAMPED SQUARE SLAB FOR OPTIMAL PLASTIC DESIGN USING 10mm ⌀ M.S. BARS.

$$c = \frac{109.16 \times 0.7854 \times 1400 \times 10.5 \times 0.87 \times 64}{600 \times 5^4 \times 100}$$

$$= 1.86 \qquad \ldots \quad (7.1)$$

7.5.2 Reinforcement Layout for the 'Uncurtailed' Solution

By varying only the spacing of bars in the finite width of slab and without curtailing the length of bars, a layout as shown in Fig. 7.7 can be found very easily.

Total length of reinforcing 10 mm Ø bars for this case will be 181.4 metres.

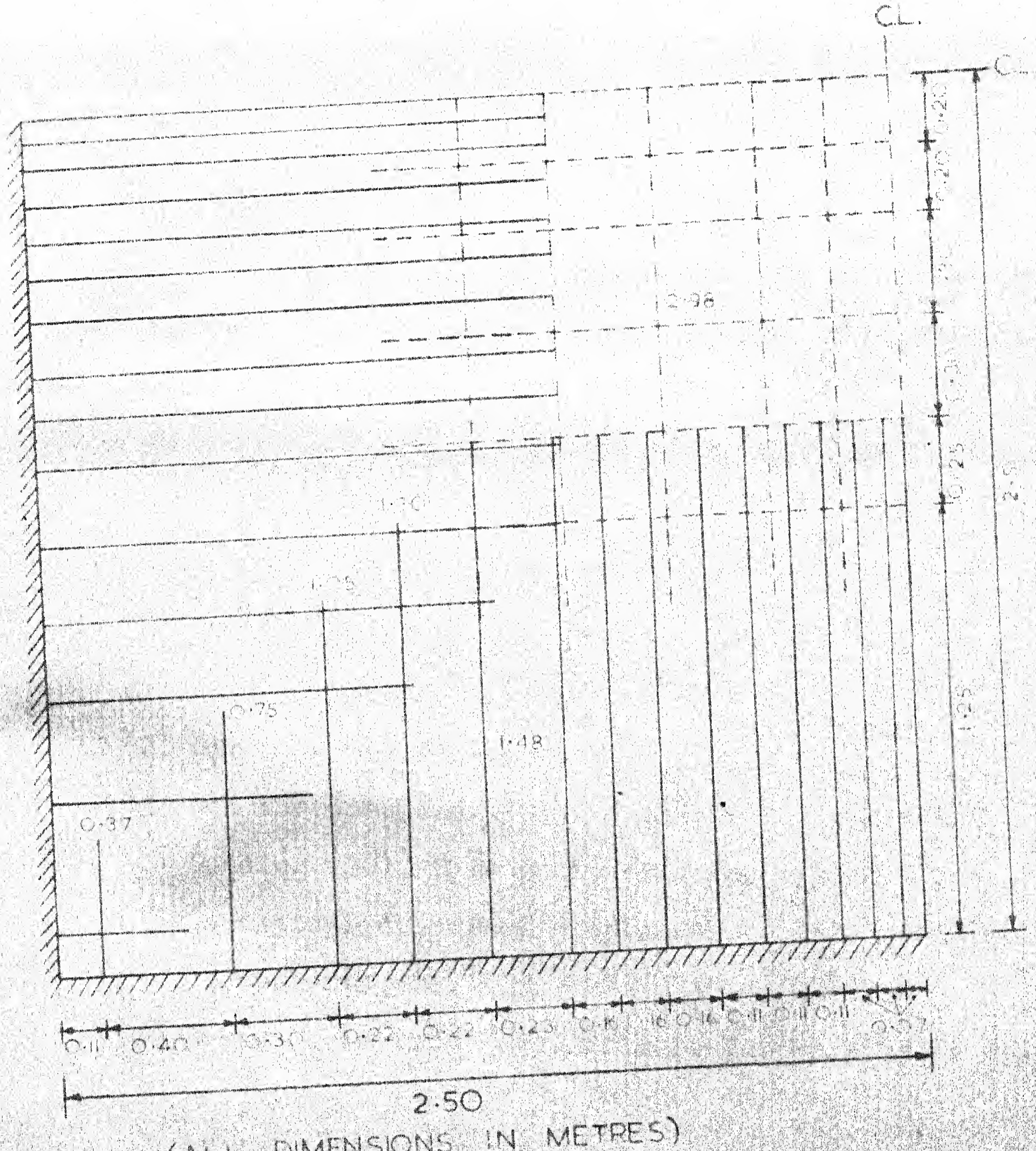
$$c = \frac{181.4 \times 0.7854 \times 1400 \times 10.5 \times 0.87 \times 64}{600 \times 5^4 \times 100}$$

$$= 3.00 \qquad \ldots \quad (7.2)$$

7.5.3 Comparative Study

Comparing with the theoretical values of c obtained in Table 9 for the 'curtailed' and 'uncurtailed' solutions, the actual and practical values are little higher due to positioning of reinforcement at the discrete points, the anchorage length required beyond the theoretical point of cut-off etc.

It may be noted that the uncurtailed reinforcement lay-out is quite practical and very easy to implement on the works with the very low number of repetitions. Uncurtailed reinforcement layout as shown in Fig. 7.7, shows considerable economy in the amount of reinforcement compared to those required by various codes of practice.



——— FOR SUPPORT MOMENTS, - - - - FOR SPAN MOMENTS

FIG. 7·7 UNCURTAILED REINFORCEMENT LAYOUT FOR THE CLAMPED SQUARE SLAB FOR OPTIMAL PLASTIC DESIGN USING 10 mm Ø M.S. BARS.

CHAPTER 8

OPTIMAL SYNTHESIS

8.1 INTRODUCTION

In this chapter, an exploratory discussion of the use of optimal synthesis of slabs for various design criteria like safety serviceability and economy simultaneously is given, so as to attempt to link the discussion in chapters 3, 4 and 5. This may lead to absolute optimal solution for reinforced concrete slabs. Various approaches to structural design problem can be divided into two branches as:

(1) Conventional design and

(2) Direct design or Structural synthesis

depending upon the nature of initial assumption regarding,

(a) Loading (LO). This includes force system, temperature variations or forced displacements like settlement of supports etc.

(b) Design Behaviour (DB). This is for working as well as ultimate loads.

(c) Geometrical properties (G) of the members of the structure, and

(d) Material properties (M) of the members of the structure.

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(c) Geometrical properties (G) of the members of the structure, and

(d) Material properties (M) of the members of the structure.

Conventional Design:

Conventional design procedure, which is a trial and analysis procedure for the exhaustive search of a solution which satisfies the design behaviour. It assumes initially 'G' and 'M'. This procedure can be divided into two sub-parts as:

(i) Behavioural Analysis

Here, the design behaviour (DB) is found by the trial and analysis, with known LO, G and M.

$$DB = DB\,(LO,\ G,\ M) \qquad \ldots \qquad (8.1)$$

(ii) Load Capacity Analysis

Here the load (LO) carrying capacity is found having known DB, G and M

$$LO = LO\,(DB,\ G,\ M) \qquad \ldots \qquad (8.2)$$

Most of the design processes fall into above two categories defined by Equations (8.1) and (8.2).

Conventional design may lead to an oversafe and uneconomical design solution, and ultimately affect the basic values of structural design, very badly.

Structural Synthesis:

Here, the initial guessing as done in the conventional design for 'G' and 'M' is eliminated, and direct design solutions for 'G' and 'M' are availed for known 'LO' and 'DB'.

$$\left.\begin{aligned} M &= M\ (DB,\ LO) \\ G &= G\ (DB,\ LO) \end{aligned}\right\} \quad \ldots \quad (8.3)$$

For the reinforced concrete slabs, material properties (M) are fixed, as the usual concrete mix used is M.150 or equivalent, and mild steel plain or high strength deformed steel bars are used. Thus, in the game of synthesis, the parameter 'M' is eliminated.

As mentioned in chapter 4, Brotchie[18] presented direct design approach for the reinforced concrete slabs, based on the uniform strength concept and using Equations (8.3). This approach had been discussed in detail in chapter 4.

Rozvany[86] suggested to carryout structural synthesis using the equilibrium equations only, such that the compatibility equations are automatically satisfied.

Optimal Synthesis:

Number of design solutions satisfying Equations (8.3) will be very high and which one to select will depend upon the criterion represented by a functional F as,

$$F = F\ (G,\ M) = \text{Min (or Max)} \quad \ldots \quad (8.4)$$

Here, the functional F may represent the cost or safety or strength or serviceability or minimum strain energy or maximum potential energy etc. The solution satisfying F, the functional,

then the optimal solution with respect to the extremum criteria for F, is reached. This entire process, which needs the rigorous approach of variational calculus, is an optimal synthesis, as defined by Rozvany[86].

Advantages of the optimal synthesis are many, like

(1) Intuitive decisions are eliminated

(2) It yields the direct solutions for the specified extremum criteria

(3) In some cases, it may be much simpler mathematical procedure than the conventional analysis.

Rozvany[86] had found solutions with respect to the strength, for the uniform thickness of the slab and varying the reinforcement from point to point, by considering the extremum criteria to be the volume of reinforcement.

Kaliszky[64] for the cost as the extremum criteria, derived that the reinforcement layout following the elastic stress field at different point is an optimal solution.

8.2 LIMITATION OF THE OPTIMAL SYNTHESIS

Limitation of the optimal synthesis as a design process, is due to the fact that the all extremum criteria (cost, strength safety serviceability etc.) can be introduced with great difficulty simultaneously to get an absolute optimal solution.

For different extremum criteria like safety, strength and economy, without the use of the variational calculus[87], optimal design solutions are presented in chapters 3, 4 and 5 respectively.

It can be seen very easily that the MHSM assures optimal safety and economy, while the strength is guaranteed by the redistribution of forces and the serviceability criterion is satisfied by the WSD, limiting the span/thickness ratio, and limits specified for crack width, deflections etc. Thus, MHSM though not based on rigorous optimization technique like optimal synthesis, is the correct step towards the safety, rationality and economy in the reinforced concrete slabs.

CHAPTER 9

CONCLUSIONS

9.1 CONCLUSIONS

The conclusions reached as a result of this study, are as follows:

1. Two-way slabs when designed according to elastic methods and tested to failure have been found exhibiting very high factor of safety. The reason for getting higher factor of safety is due to many factors like neglecting the redistribution of forces, specifying uniform moment values equal to maximum positive or negative moments found at critical points under worst combination (very hypothetical) of loads, etc.

2. The ductility and the inherent reserve strength of slab are not taken into account in methods based on pure elastic analysis, which form the basis of most codes of practices.

3. In elastic analysis live load effect is taken into account by putting live load in different panels in form of pattern loading to get worst combination for moments. In slabs live load is also acting the direction of the dead load. With inelastic analysis there is no need for separate and special study for live load effects. This simplifies the problem of analysis and design, considerably.

4. MHSM presented in chapter 3, takes the advantage of the potential strength of slabs and yields moment coefficients

are economical than those given in IS:456-1964 and ACI:318-63.

5. With the use of the MHSM, the behaviour at the working load is guaranteed by following factors.

(a) Use of inelastic analysis-elastic design (overall thickness required by the WSD is quite more than by the ultimate strength theory for same ultimate moment carrying capacity).

(b) Stiffness requirements of the codes (to guard against excessive deflections at working loads, codes specify limit on span/thickness ratio to assure minimum thickness ratio to assure minimum thickness required).

(c) Limit specified on crack width (usually 0.15 mm).

(d) Neglecting tension in the concrete (WSD neglects tension in the concrete).

(e) Allowing temporary marginal increase in stresses under the rare occurance of the designed magnitude of live load, and its pattern loading. (Subject to limits specified in (c) above is also not violated).

6. With the use of MHSM, the factor of safety of the two-way slabs is brought to the parity with the other flexural members like one-way slabs, flat slabs etc.

7. The inelastic analysis,-elastic design approach of MHSM can be used for slabs with high strength deformed bars. With the structural lightweight concrete, MHSM can be used with the caution for cracking and deflection.

8. Before full-scale tests on the two-way slabs designed according to MHSM, are carried out, in future revisions of IS: 456-1964, present moment coefficients (of Method 2) may be changed in the direction towards recommanded moment coefficients in Tables 3 and 4, such that at least the partial advantage of ductility and the resulting economy is gained.

9. MHSM neglects the advantages of the membrane action and strain-hardening effects. In U.S.S.R. membrane action is considered and the collapse load increased by 10 percent or 20 percent depending upon the type of the panel or for the same collapse load moments are reduced by 10 percent or 20 percent depending upon the type of the panel. This emphasises that even with the rational design procedure like MHSM, there is still conservatism due to prevailing ignorance of the proper yield criteria.

10. In general, a saving of 15 percent to 20 percent in the total reinforcement is possible with MHSM, with an overall savings of 8 percent to 10 percent approximately.

11. Use of MHSM is not recommended for the unusual applications such as

(a) Concentrated loads,

(b) Live loads of sufficiently high values,

(c) Hydrostatic loads,

(d) Significantly high percentage of reinforcement in the slab, and

(e) Slab with the part of the boundary free to deflect and rotate.

12. The minimum reinforcement layouts in the slabs either by the elastic theory or the optimum plastic design, using orthogonal straight (in some cases 'non-orthogonal layouts have shown considerably less volume of reinforcement) reinforcement, are practical only for the case of highly repetitive nature or the trade off between the saving of material and the increase in the fabrication cost is justifiable.

13. Use of 'Isomoms' can be made to derive 'uncurtailed' or 'curtailed' reinforcement layouts using the elastic theory.

14. MHSM offers an essential feature for the economy of material, a curtailment of the reinforcement (not to the extent of 'uncurtailed' or 'curtailed' solutions) as a part of the design procedure.

15. Using the moment coefficients specified by either the codes or MHSM, and then designing by WSD, for the given concrete mix (for slabs it is usually M.150) and the type of reinforcing steel, the minimum cost solution is the one obtained when the assumed and the required overall thicknesses match with each other. In this process, the requirements of the code for the minimum thickness, minimum percentage of reinforment, minimum cover and maximum spacing of bars; are satisfied. This conclusion is valid for Indian conditions. For other countries, this has to be verified.

16. To derive an optimum cost solution, classical method of differential calculus fails miserably for the slabs with

complex boundary conditions (other than 'all sides simply supported').

17. The minimum cost solution obtained in chapter 5, using moment coefficients and WSD, is not the global but the restricted one. Global solution can be had by trying out the various combination of different type of concrete and steel; using both the WSD and ultimate strength theory.

18. Two-way slabs when supported by the re-entrant corners, intermediate column or wall support; with clear soffit, when designed by Hillerborg's approximate theory of elasticity involves more cost than the slab with the beams.

19. MHSM and the minimum cost solution using it, assure the 'near' optimal solution, considering the process of the optimal synthesis.

9.2 SUGGESTIONS FOR FUTURE RESEARCH

To understand the behaviour of the R.C. slabs more precisely and to derive optimal solutions for various extremum criteria, the work in the following areas is needed:

1. Global minimum cost design of the two-way R.C slabs with different boundary conditions, using the WSD, by trying out various combinations of all concrete mixes i.e. M.150, M.200, etc. and different varieties of steel.

2. Optimal synthesis using all extremum criteria like safety, strength, economy, serviceability etc. simultaneously

to search for an absolute optimal solution.

3. For individual criteria like safety optimal synthesis based on rigorous mathematical formulation, an optimal solution (may be better than MHSM) is required.

4. Full-scale tests on the two-way slabs designed according to MHSM are needed especially to study the behaviour at working loads.

5. Proper yield criterion for the R.C. slabs incorporating twisting moment, membrane action and strain-hardening is required to understand the behaviour, exactly.

6. Behaviour under dynamic forces such as earthquake forces, blast forces etc. of the two-way R.C. slabs with the minimum reinforcement layout discussed in chapter 4, for different boundary conditions is necessary.

7. Study of 'Isomoms' for the two-way R.C. slabs according to the elastic theory, for all boundary conditions and span ratios is necessary to specify the practical reinforcement layout with strong and weak bands.

8. Full-scale tests on the clamped square slab with the absolute minimum (i.e. curtailed solution using Hillerborg's strip method) reinforcement layout is necessary to compare the behaviour with that of optimized simply supported square slab by Rozvany.

9. Considering environmental effects such as creep, shrinkage, temperature differentials etc, as a governing criteria, optimal design solutions for two-way rectangular slabs are desired.

APPENDIX A

NOTATIONS

A.1 ALPHABETIC SYMBOLS

A	Cross-sectional area (or in some cases constant)
a, b	Width and length of a rectangular plate in x and y directions respectively
A_{st}	Area, of steel per unit width of slab
A_{stx}, A_{sty}, A_{stpx}, A_{stpy}	Areas of steel for positive and negative moments in x and y direction
A_{stmax}	Maximum area of steel in the slab
B	Unit width of slab (or constant in some cases)
$\overline{B}$	Constant
C	Initial cost of slab per unit area (or a dimension specified in ACI: 318-63 for flat slabs)
c, $\overline{c}$	Constants indicating the total volume of reinforcements, in the simply supported and clamped, square slabs respectively.
c_1	Strip length from support to the point of maximum span moment.
C_c, C_s	Unit costs of concrete and steel, respectively
$\overline{C}_c$	Parameter defining cost of concrete
C_f, C_s, C_p	Moment coefficients for span, support and re-entrant corner (or column or wall) respectively.

C_x, C_y, C_{Nx}, C_{Ny}	Moment coefficients for positive and negative design moments in x and y directions for trapezoidal moment distribution.
C_w	Crack width
D	Flexural Stiffness of plate
D_b	Bar diameter
d	Effective depth of the slab
DB	Design behaviour
E	Modulus of elasticity of plate material
F	Functionals (with respect to cost, safety, serviceability etc.)
$f_m(y)$, $f_k(y)$	Functions in y
G	Geometric properties
h	Overall thickness of plate
j	Lever-arm factor
J_1,J_2	Constants
K	Constant (a ratio of lever-arm factors at ultimate load and load at which yielding starts)
$\bar{K}$	Coefficient depending upon the probability
k	Curvature
$\bar{k}$	Ratio of overall thickness to effective depth, of slab.
k_1,k_2,k_3,k_4	Parameters defining support moments with respect to moment M in x direction.
k_x,k_y	Curvatures in x and y directions respectively

L	Span of simply supported or clamped or interior panel of flat slab; square in shape.
l	Strip span
LO	Generalised loading
l_x, l_y	Effective spans in x and y directions respectively
M	Moment in the slab (in some cases in x direction)
M', M^F	Support moments without and with complete restraint
M_o, M_1, M_2	Moment carrying capacity of the slab defined by various yield criteria, and principal moments, respectively
M_{cr}	Moment carrying capacity at the time of cracking
M_f, M_s, M_p	Span, support and re-entrant corner (or column or wall) moments respectively
M_{max}	Maximum moment
M_n, M_{nt}	Normal and twisting moments for the yield-line
M_u	Ultimate moment carrying capacity
M_{ux}, M_{uy}	Ultimate moment carrying capacities in x and y directions respectively
$M_{working}$ or M_w	Design moment at working load
M_x, M_y, M_{xy}	Moments in x and y directions; and twisting moment in xy plane respectively.
M'_x, M'_y	Support moments in x and y directions respectively
$M*_x$, $M*_y$	Principal moments (constrained) in x and y directions respectively

m_x, m_y, m'_x, m'_y Moment coefficients for span and support design moments in x and y directions respectively; for uniform moment distribution.

M_{yield} Moment carrying capacity of slab, when steel has yielded

N_{ij} Coefficients

n Neutral axis factor at the time of cracking

n_1 Neutral axis factor for balanced section (WSD)

P, P_x, P_{max} Percentage of reinforcement and corresponding values in x direction and maximum, respectively.

$P_{x\ opt}$, $P_{y\ opt}$ Optimum percentages of reinforcement in x and y directions, respectively.

q, q_w, q_u Uniformly distributed load over panel per unit area, and corresponding values at working and ultimate loads

Q^* Design value for load

Q_c, Q_s Quantities of concrete and steel, per unit area of the slab, respectively

Q_k Characteristic value of the load

Q_m Value of the most unfavourable loading with 50 percent probability of its being exceeded, upto abnormally high value, once in the expected life of the structure

q', q'_{max} Parameter governing the behaviour and its maximum value respectively

$Q_{s\ min}$	Minimum amount of steel per unit area of slab
R	Parameter defined as $= \frac{\sigma_{cb}\, n_1}{2} \left(1 - \frac{n_1}{3} \right)$
R_1, R_2	Regions as defined by Rozvany
R_x, R_y	Regions defined in Hillerborg's strip method for transfer of load
R_{xy}	Region of the plate where load is distributed in both x and y directions.
S, AL	Anchorage Length
s	Span ratio $= l_y / l_x$
S_w	Total reinforcement per panel
t	Effective thickness of slab
t_o, t_{in}, t_{ove}	Overall thickness of the slab at various stages in iterative procedure
t_{min}	Minimum overall thickness required from stiffness considerations
U	Factor of safety (w.r. to Collapse load)
V	Total moment volume for the structure
V_m	Total volume of reinforcement for simply supported square slab
V_s	Total volume of reinforcement for clamped square slab.
W	Uniformly distributed load over fixed beam, per unit length
w (x,y)	Deflection function
X,Y	Parameters defining transformed dimensions of the orthotropic slab

$X_1, X_2, X_3, ..$	Strips in x direction
$x, \bar{x}$	Variables in x directions
x, y	Cartesian coordinates in x-y plane
x_1, x_2	Lengths defining the mode of collapse for restrained rectangular slab.
$Y_1, Y_2, Y_3, ..$	Strips in y direction
$Y_m(y), Y_{mh}(y), Y_{mp}(y)$	Hyperbolic function in y; and its homogeneous and particular parts, respectively

A.2 GREEK SYMBOLS

α, β, γ	Parameters defining curtailment of bars
$\bar{\alpha}$	Constant
α_x	Coefficient defined by Method 2 of IS:456-1964.
γ_m	Safety factor for material strength
γ_s	Safety factor for loads
δ	Relative mean quadratic deviation
λ	Aspect ratio for simply supported rectangular slabs
μ	Coefficient of Orthotropy (i.e. ratio of span moments in y and x directions)
ν	Poisson's ratio
ρ	Radious of Curvature
σ^*	Design strength of material
σ_{cb}	Permissible bending stress in concrete, in compression

σ_{cu}	Cube strength of concrete (15 cms cubes at the rate of 28 days)
σ_k	Characteristic strength of material
σ_m	Arithmatic mean of various experimental results for material strength
σ_{st}	Permissible tensile stress in steel
σ_{sy}	Yield stress in steel
τ, τ_a	Punching shear stress and allowable shear stress in concrete, respectively
ϕ	Ratio of M' and M^F; defined for handling cases of partial fixity
$\phi_m(x)$	Function in x
ϕ_s, ϕ_p	Parameter ϕ for support and re-entrant corner (or column or wall), respectively

A.3 ABBREVIATIONS

ACI	American Concrete Institute, Detroit, Mich., USA
MHSM	Modified Hillerborg's Strip Method
NBO	National Building Organization, New Delhi, India
USD	Ultimate Strength Design
WSD	Working (or permissible) Stress Design

APPENDIX B

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APPENDIX C

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APPENDIX D

FORMULAE FOR ANALYSING RECTANGULAR SLABS WITH RESTRAINED EDGES AND ORTHOTROPIC REINFORCEMENT

For type of slabs shown in Fig. D.1, the equilibrium method (segment equilibrium) as suggested by Ingerslav[28] originally, is seen best compared to the energy methods which involve laborious algebra.

The equilibrium method can be used for the type slabs shown in Fig. D.1 due to the fact that[53]

'Knot-forces vanish where positive lines converge, and also to the known values of edge-forces where fracture lines meet free edges.'

Problem concerned here, do not involve free edges (free to deflect and rotate) and, as such the equilibrium method will be used now onwards.

Ultimate moment in x direction can be found (detailed derivations can be found elsewhere[53]) as:

$$M_u = \frac{q_u \, l_x^2 \, s^2}{6 \, Y^2} \left| \sqrt{3 + \left(\frac{sX}{Y} \right)^2} - \frac{sX}{Y} \right|^2 \qquad \ldots \qquad (D.1)$$

where X and Y are defined below in Equations (D.5) and (D.7) for different type of moment distribution across the panel. Equation (D.1) is valid for:

$$x_1 + x_2 \leq l_x \qquad \ldots \qquad (D.2)$$

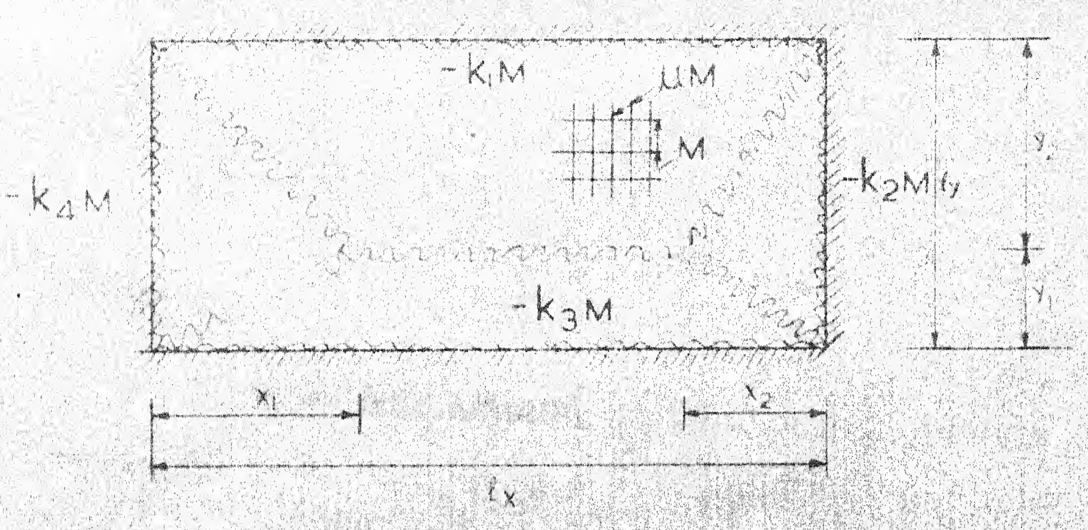

FIG. D.1 RECTANGULAR SLAB WITH RESTRAINED EDGES AND ORTHOTROPIC REINFORCEMENT

If

$$x_1 + x_2 > l_x \qquad \ldots \qquad (D.3)$$

then, the selected failure mode indicated in Fig. D.1, is not valid, but a similar pattern, turned through 90° will occur and all quantities must be re-oriented to suit the new direction.

In above Equations (D.2) and (D.3) x_1 and x_2 are defined below in Equations (D.6) and (D.8) for different type of moment distribution across the panel.

Now, to compute the factor of safety, use of Equation (3.1) will be made. In Equation (3.1), $M_{working}$ can be found for the design (working) load using the moment coefficients specified in the codes or MHSM. M_{yield} (= M_u) can be found from Equation (D.1). The factor of safety can now be found as

$$= \frac{q_u}{q_w} = \frac{\sigma_{sy}}{\sigma_{st}} K \frac{6\, Y^2 \text{ (Moment coefficients of code or MHSM)}}{l_x^2\, s^2 \left[\sqrt{3 + \left(\frac{sX}{Y}\right)^2} - \frac{sX}{Y}\right]^2} \qquad \ldots \qquad (D.4)$$

Values of X, Y, x_1 and x_2:

(a) For uniform moment distribution - no provision of the torsion reinforcement

This case resembles to Method 2 of IS:456-1964 without any provision of the torsion reinforcement (Fig. D-1).

$$\left.\begin{aligned} X &= \sqrt{1 + k_2} + \sqrt{1 + k_4} \\ Y &= \sqrt{\mu + k_1} + \sqrt{\mu + k_3} \end{aligned}\right\} \quad \ldots \qquad (D.5)$$

and

$$\left.\begin{aligned} x_1 &= \sqrt{6\,M\,(1 + k_4)} \\ x_2 &= \sqrt{6\,M\,(1 + k_2)} \end{aligned}\right\} \quad \ldots \qquad (D.6)$$

where k_1, k_2, k_3, k_4 are the ratios of support moments to the moment in x direction, for different edges.

and is the coefficient of orthotropy (ratio of the moment in y direction to the moment in the x direction)

(b) For Trapezoidal moment distribution - torsion reinforcement provided.

This case resembles to the ACI: 318-63 code provisions.

$$\left.\begin{aligned} X &= \sqrt{\frac{5}{6}\,(1 + k_2) + \frac{\mu(N_{23} + N_{21})}{5\,s}} \\ &\quad + \sqrt{\frac{5}{6}\,(1 + k_4) + \frac{\mu(N_{41} + N_{43})}{5\,s}} \\ Y &= \sqrt{\frac{5}{6}\,(\mu + k_1) + \frac{\mu(N_{14} + N_{12})}{5}} \\ &\quad + \sqrt{\frac{5}{6}\,(\mu + k_3) + \frac{\mu(N_{34} + N_{32})}{5}} \end{aligned}\right\} \quad \ldots \qquad (D.7)$$

and

$$\left.\begin{aligned} x_1 &= \sqrt{5M\,(1 + k_4) + \frac{6\mu M}{5s}\,(N_{41} + N_{43})} \\ x_2 &= \sqrt{5M\,(1 + k_2) + \frac{6\mu M}{5s}\,(N_{23} + N_{21})} \end{aligned}\right\} \qquad (D.8)$$

where $N_{23} = N_{21} = N_{41} = N_{43} = N_{14} = N_{12} = N_{34} = N_{32} = 0.707$

Assumptions made in deriving Equations (D.7) and (D.8) have been discussed in detail in 3.4.2 (Fig. 3.8).